WEYL GEOMETRY IN LATE 20TH CENTURY PHYSICS

ERHARD SCHOLZ

Abstract. Weyl's original scale geometry of 1918 ("purely infinitesimal geometry") was withdrawn from physical theory in the early 1920s. It had a comeback in the last third of the 20th century in different contexts: scalar tensor theories of gravity, foundations of physics (gravity, quantum mechanics), elementary particle physics, and cosmology. Here we survey the last two segments. It seems that Weyl geometry continues to have an open research potential for the foundations of physics after the turn of the century.

1. Introduction

Roughly at the time when his famous book *Raum · Zeit · Materie* (RZM) went into print, Hermann Weyl generalized Riemannian geometry by introducing scale freedom of the underlying metric, in order to bring a more basic "purely infinitesimal" point of view to bear (Weyl 1918*c*, Weyl 1918*a*). How Weyl extended his idea of scale gauge to a unified theory of the electromagnetic and gravitational fields, how this proposal was received among physicists, how it was given up – in its original form – by the inventor already two years later, and how it was transformed into the now generally accepted $U(1)$-gauge theory of the electromagnetic field, has been extensively studied.[1] Many times Weyl's original scale gauge geometry was proclaimed dead, physically misleading or, at least, useless as a physical concept. But it had surprising come-backs in various research programs of physics. It seems well alive at the turn to the new century.

Weylian geometry was taken up explicitly or half-knowingly in different research fields of theoretical physics during the second half of the 20th century (very rough time schedule):

- 1950/60s: Jordan-Brans-Dicke theory
- 1970s: a double retake of Weyl geometry by Dirac and Utiyamah
- 1970/80s: Ehlers-Pirani-Schild and successor studies
- 1980s: geometrization of (de Broglie Bohm) quantum potential
- 1980/90s: scale invariance and the Higgs mechanism
- 1990/2000s: scale covariance in recent cosmology

Date: 23. 09. 2009 .

[1](Vizgin 1994, Straumann 1987, Sigurdsson 1991, Goenner 2004, O'Raifeartaigh 2000, O'Raifeartaigh 1997, Scholz 2001, Scholz 2004, Scholz 2005*a*).

All these topics are worth of closer historical studies. Here we concentrate on the last two topics. The first four have to be left to a more extensive study.

With the rise of the standard model of elementary particles (SMEP) during the 1970s a new context for the discussion of fundamental questions in general relativity formed.[2] That led to an input of new ideas into gravity. Two subjects played a crucial role for our topic: *scale* or *conformal invariance* of the known interactions of high energy physics (with exception of gravity) and the intriguing idea of *symmetry reduction* imported from solid state physics to the electroweak sector of the standard model. The latter is usually understood as symmetry breaking due to some dynamical process (Nambu, Goldstone, Englert, Higgs, Kibble e.a.). The increasingly successful standard model worked with conformal invariant interaction fields, mathematically spoken connections with values in the Lie algebras of "internal" symmetry groups (i.e., unrelated to the spacetime), $SU(2) \times U(1)_Y$ for the electroweak (ew) fields, $SU(3)$ for the chromodynamic field modelling strong interactions, and $U(1)_{\mathrm{em}}$ for the electromagnetic (em) field, inherited from the 1920s. In the SMEP electromagnetism appears as a residual phenomenon, after breaking the isospin $SU(2)$ symmetry of the ew group to the isotropy group $U(1)_{\mathrm{em}}$ of a hypothetical vacuum state. The latter is usually characterized by a *Higgs field* Φ, a "scalar" field (i.e. not transforming under spacetime coordinate changes) with values in an isospin $\frac{1}{2}$ representation of the weak $SU(2)$ group. If Φ characterizes dynamical symmetry breaking, it should have a massive quantum state, the Higgs boson (Higgs 1964, Weinberg 1967). The whole procedure became known under the name "Higgs mechanism".[3]

Three interrelated questions arose naturally if one wanted to bring gravity closer to the physics of the standard model:

(i) Is it possible to bring conformal, or at least scale covariant generalizations of classical (Einsteinian) relativity into a coherent common frame with the standard model SMEP?[4]

(ii) Is it possible to embed classical relativity in a quantized theory of gravity?

[2]It is complemented by the standard model of cosmology, SMC. Both, SMC and SMEP, developed a peculiar symbiosis since the 1970s (Kaiser 2007). Strictly speaking, *the* standard model without further specifications consists of the two closely related complementary parts SMEP and SMC.

[3]Sometimes called in more length and greater historical justness "Englert- Brout-Higgs-Guralnik-Hagen-Kibble" mechanism.

[4]Such an attempt seemed to be supported experimentally by the phenomenon of (Bjorken) scaling in deep inelastic electron-proton scattering experiments. The latter indicated, at first glance, an active scaling symmetry of mass/energy in high energy physics; but it turned out to hold only approximatively and of restricted range.

(iii) Or just the other way round, can "gravity do something like the Higgs"?[5] That would be the case if the mass acquirement of electroweak bosons could be understood by a Brans-Dicke like extension of gravitational structures.

These questions were posed and attacked with differing degrees of success since the 1970s to the present. Some of these contributions, mostly referring to questions (i) and (iii), were closely related to Weylian scale geometry or even openly formulated in this framework. The literature on these questions is immense. Obviously we can only scratch on the surface of it in our survey, with strong selection according to the criterion given by the title of this paper. So we exclude discussion of topic (ii), although it was historically closer related to the other ones than it appears here (section 3).

In the last three decades of the 20th century a dense cooperation between particle physics, astrophysics and cosmology was formed. The emergence of this intellecual and disciplinary symbiosis had many causes; some of them are discussed in (Kaiser 2007) and by C. Smeenk (this volume). Both papers share a common interest in inflationary theories of the very early universe. But in the background of this reorganization more empirically driven changes, like the accumulating evidence for "dark matter" by astronomical observations in the 1970s, were surely of great importance (Rubin 2003, Trimble 1990). That had again strong theoretical repercussions. In the course of the 1980/90s it forced astronomers and astrophysicists to assume a large amount of non-visible, non-baryonic matter with rather peculiar properties. In the late 1990s increasing and different evidence spoke strongly in favour of a non-vanishing cosmological constant Λ. It was now interpreted as a "dark energy" contribution to the dynamics of the universe (Earman 2001).

The second part of the 1990s led to a relatively coherent picture of the *standard model of cosmology* SMC with a precise specification of the values of the energy densities Ω_m, Ω_Λ of (mostly "dark") matter and of "dark" energy as the central parameters of the model. This specification depended, of course, on the choice of the Friedman-Lemaitre spacetimes as theoretical reference frame. Ω_m and Ω_Λ together determine the adaptable parameters of this model class (with cosmological constant). The result was the now favoured ΛCDM model.[6] In this sense, the geometry of the physical universe, at least its empirically accessible part, seems to be well determined, in distinction to the quantitative underdetermination of many of the earlier cosmological world pictures of extra-modern or early modern cultures (Kragh 2007). But the new questions related to "dark matter" and "dark energy" also induced attempts for widening the frame of classical GRT. Scale

[5]Formulation due to (Pawłowski 1990).

[6]CDM stands for for cold dark matter and Λ for a non-vanishing cosmological constant.

covariant scalar fields in the framework of conformal geometry, Weyl geometry, or Jordan-Brans-Dicke theory formed an important cluster of such alternative attempts. We shall have a look at them in section (4). But before we enter this discussion, or pose the question of the role of scale covariance in particle physics, we give a short review of the central features of Weyl geometry, and its relation to Brans-Dicke theory, from a systematic point of view. Readers with a background in these topics might like to skip the next section and pass directly to section (3). The paper is concluded by a short evaluation of our survey (section 5).

2. Preliminaries on Weyl geometry and JBD theory

Weylian metric, Weyl structure. A Weylian metric on a differentiable manifold M (in the following mostly $dim\ M = 4$) can be given by pairs (g, φ) of a non-degenerate symmetric differential two form g, here of Lorentzian signature $(3,1) = (-,+,+,+,)$, and a differential 1-form φ. The *Weylian metric* consists of the equivalence class of such pairs, with $(\tilde{g}, \tilde{\varphi}) \sim (g, \varphi)$ iff

$$(1) \qquad (i) \quad \tilde{g} = \Omega^2 g\ , \qquad (ii) \quad \tilde{\varphi} = \varphi - d \log \Omega$$

for a strictly positive real function $\Omega > 0$ on M. Chosing a representative means to *gauge* the Weylian metric; g is then the *Riemannian component* and φ the *scale connection* of the gauge. A change of representative (1) is called a *Weyl* or *scale transformation*; it consists of a conformal rescaling (i) and a *scale gauge transformation* (ii). A manifold with a Weylian metric $(M, [g, \varphi])$ will be called a *Weylian manifold.* For more detailed introductions to Weyl geometry in the theoretical physics literature see (Weyl 1918*b*, Bergmann 1942, Dirac 1973), for mathematical introductions (Folland 1970, Higa 1993).

In the recent mathematical literature a *Weyl structure* on a differentiable manifold M is specified by a pair (c, ∇) consisting of a conformal structure $c = [g]$ and an affine, i.e. torsion free, connection Γ, respectively its covariant derivative ∇. The latter is constrained by the property that for any $g \in c$ there is a differential 1-form φ_g such that

$$(2) \qquad \nabla g + 2\varphi_g \otimes g = 0\,,$$

(Calderbank 2000, Gauduchon 1995, Higa 1993, Ornea 2001). We shall call this *weak compatibility* of the affine connection with the metric.[7] One could also formulate the compatibility by

$$(3) \qquad \Gamma - {}_g\Gamma = 1 \otimes \varphi_g + \varphi_g \otimes 1 - g \otimes \varphi_g^*\,,$$

where 1 denotes the identiy in $Hom(V, V)$ for every $V = T_xM$, φ_g^* is the dual of φ_g with respect to g, and ${}_g\Gamma$ is the Levi-Civita conection of

[7]Physicists usually prefer to speak of a “semimetric connection”' (Hayashi/Kugo 1977) or even of a “nonmetricity” of the connection (Hehl e.a. 1995) etc.

g. Written in coordinates that means

$$\Gamma^{\mu}_{\nu\lambda} = {}_g\Gamma^{\mu}_{\nu\lambda} + \delta^{\mu}_{\nu}\varphi_{\lambda} + \delta^{\mu}_{\lambda}\varphi_{\nu} - g_{\nu\lambda}\varphi^{\mu} \, , \tag{4}$$

if ${}_g\Gamma^{\mu}_{\nu\lambda}$ denote the coefficients of the affine connection with respect to the Riemannian component g only.

This is just another way to specify the structure of a Weylian manifold, because $[(g, \varphi)]$ is compatible with exactly one affine connection. (8) is the condition that the scale covariant derivative of g vanishes in every gauge (see below). The Weyl structure is called *closed*, respectively *exact*, iff the differential 1-form φ_g is so (for any g). In agreement with large parts of the physics literature on Weyl geometry, we shall use the terminology *integrable* in the sense of closed, i.e., in a local sense.

In some part of the physics literature a *change of scale* like in (1 (i)) is considered without explicitly mentioning the accompanying *gauge transformation* (ii). Then a scale transformation is identified with a conformal transformation of the metric. That may be misleading but need not, if the second part of (1) is respected indirectly. In any case we have to distinguish between a *strictly conformal* point of view and a *Weyl geometric* one. In the first case we deal with $c = [g]$ only, in the second case we refer to the whole Weyl metric $[(g, \varphi)]$, respectively Weyl structure (c, ∇).[8]

Covariant derivative(s), curvature, Weyl fields. The covariant derivative with respect to Γ will be denoted (like above) by ∇. The covariant derivative with respect to the Riemannian component of the metric only will be indicated by ${}_g\nabla$. ∇ is an invariant operation for vector and tensor fields on M, which are themselves invariant under gauge transformations. The same can be said for *geodesics* γ_W of Weylian geometry, defined by ∇, and for the *Riemann curvature* tensor $Riem = (R^{\alpha}_{\beta\gamma\delta})$ and its contraction, the Ricci tensor $Ric = (R_{\mu\nu})$. The contraction is defined with respect to the 2nd and 3rd component

$$R_{\mu\nu} := R^{\alpha}_{\mu\alpha\nu} \tag{5}$$

.

Functions or (vector, tensor, spinor ...) fields F on M, which transform under gauge transformations like

$$F \longmapsto \tilde{F} = \Omega^k F \, . \tag{6}$$

will be called *Weyl functions* or *Weyl fields* on M of (scale or Weyl) *weight* $w(f) := k$. Examples are: $w(g_{\mu\nu}) = 2$, $w(g^{\mu\nu}) = -2$ etc. As the

[8] Both approaches work with the "localized" (physicists' language) *scale extended Poincaré group* $\mathcal{W} = \mathbb{R}^4 \ltimes SO^+(3,1) \times \mathbb{R}^+$ as gauge automorphisms. The *transition* from a strictly conformal approach to a Weyl geometric one *has nothing to do with a group reduction* (or even with "breaking" of some symmetry); it rather consists of an enrichment of the structure while upholding the automorphism group.

curvature tensor *Riem* of the Weylian metric and the Ricci curvature *Ric* are scale invariant, scalar curvature

$$R := g^{\mu\nu} R_{\mu\nu}$$

is of weight $w(R) = -2$. For the sake of historical precision it has to be noted that Weyl himself considered g to be of weight $\overline{w}(g) = 1$. Accordingly Weyl's original weights, and those of a considerable part of the literature, are half of ours, $\overline{w} = \frac{1}{2} w$. Moreover, in most of the physics literature the sign convention for the scale connection is different; both together means that a differential form $\kappa = -2\varphi$ is used in the description of Weyl geometry.[9]

The covariant derivative ∇ of Weyl fields F of weight $w(F) \neq 0$ does not lead to a scale covariant quantity. This is a deficiency of the geometric structure considered so far, if one works in a field theoretic context. It can be repaired by introducing a *scale covariant derivative* D of Weyl fields in addition to the scale invariant ∇:

$$DF := \nabla F + w(F) \varphi \otimes F \, . \tag{7}$$

A scale covariant vector field F^ν, e.g., has the scale covariant derivative

$$D_\mu F^\nu := \partial_\mu F^\nu + \Gamma^\nu_{\mu\lambda} F^\lambda + w(F) \, \varphi_\mu F^\nu \, ,$$

with the abbreviation $\partial_\mu := \frac{\partial}{\partial x^\mu}$ etc. The compatibility condition in the definition of a Weyl structure (2) can now be written as

$$Dg = 0 \, . \tag{8}$$

Relation to Jordan-Brans-Dicke theory. Jordan-Brans-Dicke (JBD) theory assumes a scalar field χ of scale weight $w(\chi) = -1$, coupled to gravity (a pseudo-Riemannian metric g) by a Lagrangian of the following type

$$\mathcal{L}_{\mathcal{JBD}}(\chi, g) = (\chi R - \frac{\omega}{\chi} \partial^\mu \chi \, \partial_\mu \chi) \sqrt{|det \, g|} \, , \tag{9}$$

with a free parameter ω and scalar curvature R. It considers conformal transformations of metric and fields, while fixing the Levi-Civita connection ∇ of the metric g underlying (9). Such a conformal rescaling is called a change of *frame*. The "original" one (defining the affine connection as the Levi-Civita connection of the Riemannian metric) like in (9) is called *Jordan frame*. The one in which the scalar field (und thus the coefficient of the Einstein-Hilbert term, the gravitational coupling coefficient) is scaled to a constant is called *Einstein frame*.

A conformal class of a metric $[g]$ and specificiation of an affine connection like in JBD theory characterizes an integrable Weyl structure. We should thus be aware that JBD theory carries the basic features

[9] Reasons for our conventions: Our sign choice of the scale connection implies positive exponent of the scale transfer function (35). Our weight convention is such that the length (norm) of vectors has weight 1.

of a Weyl geometric structure, even though most of the workers in the field do not look at it from this point of view. In this sense I consider JBD theory as a research field in which Weyl geometry stood in the background "half- knowingly".[10] Jordan and Einstein frame are nothing but Riemann gauge, respectively scalar field gauge, in the language just introduced for integrable Weyl geometry.

More recent literature, like the excellent monographs (Fujii/Maeda 2003, Faraoni 2004), often prefers a slightly different form of the scalar field and the Lagrangian, $\phi = \sqrt{2\xi^{-1}\chi}$ (scale weight $w(\phi) = -1$), $\xi = \frac{\epsilon}{4\omega}$. Then the Lagrangian acquires the form

$$\mathcal{L}_{BD} = \left(\frac{1}{2}\xi\phi^2 R - \frac{1}{2}\epsilon\partial^\mu\phi\,\partial_\mu\phi + L_{mat} \right) \sqrt{|det\, g|}\,, \tag{10}$$

where $sig\, g = (3,1) \cong (- + + +)$ and $\epsilon = \pm 1$ or 0 (Fujii/Maeda 2003, 5).[11] Penrose (1965) showed that $\mathcal{L}_{BD}$ is conformal invariant for $\xi = \frac{n-2}{4(n-1)}$ (n spacetime dimension).

Moreover in the recent literature strong arguments have accumulated to prefer Einstein gauge over Jordan gauge (Faraoni e.a. 1998). An obvious argument comes from the constraints of the coefficient ω arising from high precision gravity observations in the solar system, if Jordan frame is considered to be "physical".[12]

3. Scale covariance in particle physics

Englert's conformal approach. Francqis Englert and coworkers studied conformal gravity as part of the quantum field program (Englert 1975). In a common paper written with the astrophysicist Edgar Gunzig and others, the authors established an explicit link to JBD theory (not to Weyl geometry). They started from a "dimensionless", i.e. scale invariant, Lagrangian for gravitation with a square curvature term of an affine connection Γ *not* bound to the metric, $\mathcal{L}_{\text{grav}} = R^2 \sqrt{|det\, g|}$ in addition to a Lagrangian matter term (Englert 1975). In consequence, the authors varied with respect to the metric g and the connection Γ *independently*.

---

[10] One need not know Weyl geometry, in order to work in the framework of such a naturally given structure; just like Moliére's M. Jourdain did not know that he had spoken prose for forty years, before he was told so by a philosopher.

[11] According to Fujii/Maeda $\epsilon = 1$ corresponds to a "normal field having a positive energy, in other words, not a ghost". $\epsilon = -1$ may look at first unacceptable because it "seems to indicate negative energy", but "this need not be an immediate difficulty owing to the presence of the nonminimal coupling." '(ibid.)

[12] See the contribution by C. Will, this volume.

Further compatibility considerations made the connection weakly metric compatible, in the sense of our equ. (2), even with an integrable scale connection (Englert 1975, equ.(7)). In this way, the approach worked in a Weyl structure, but the authors did not care about it. They rather tried to be as "conformal" as possible.

In an attempted "classical phenomenological description" they characterized a pseudo-Riemanian Lagrangian of a scalar field coupled to gravity like in our equ. (10), with the necessary specification $\xi = \frac{1}{6}$ in order to achieve conformal symmetry. The scalar field was called "dilaton" and considered as a "Nambu-Goldstone boson" of a "dynamical symmetry breakdown" of the scale symmetry, but *without* a massive "scalar meson" (Englert 1975, 75). The terms corresponding to the Weylian scale connection (re-reading their paper in the light of Weyl geometry) were not considered as a physical field, but as a mathematical artefact of the analysis.

In one of the following papers Englert, now with other coauthors, studied the perturbative behaviour of conformal gravity ($\xi = \frac{n-2}{4(n-1)}$) coupled to massless fermions and photons in $n \geq 4$ dimensions.[13] They came to the conclusion that anomalies arising in the calculations for non-conformal actions disappeared at the tree and 1-loop levels in their approach. They took this as an indicator that gravitation might perhaps arise in a "natural way from spontaneous breakdown of conformal invariance" (Englert 1975, 426).

Smolin introduces Weyl geometry. Englert's e. a. paper was one of the early steps into the direction (i) of our introduction. Other authors followed and extended this view, some of them explicitly in a Weyl geometric setting, others clothed in the language of conformal geometry. The first strategy was chosen by Lee Smolin in his paper (Smolin 1979). In section 2 of the paper he gave an explicit and clear introduction to Weyl geometry. [14] The "conformally metric gravitation", as he called it, was built upon a matter-free Lagrangian with Weyl geometric curvature terms R, $Ric = (R_{\mu\nu})$, $f = (f_{\mu\nu})$ for scale curvature alone, and scale covariant Weylian derivatives D (in slight adaptation of notation):

$$(11) \qquad |det\, g|^{-\frac{1}{2}} \mathcal{L}_{\text{grav}} = -\frac{1}{2} c\, \phi^2 R + [-e_1 R^{\mu\nu} R_{\mu\nu} - e_2 R^2] + \frac{1}{2} D^\mu \phi\, D_\mu \phi - \frac{1}{4g^2} f_{\mu\nu} f^{\mu\nu} - \lambda \phi^4$$

[13] The motivation to consider $n \geq 4$ was dimensional regularization.

[14] In his bibliography he went back directly to (Weyl 1922) and (Weyl 1918*a*); he did not quote any of the later literature on Weyl geometry.

with coupling coefficients c, e_1, e_2, g, λ.[15] For coefficients of the quadratic curvature terms (in square brackets) with $e_2 = -\frac{1}{3}e_1$, the latter was variationally equivalent (equal up to divergence) to the squared conformal curvature $C^2 = C_{\mu\nu\kappa\lambda}C^{\mu\nu\kappa\lambda}$.[16]

Smolin introduced the scalar field ϕ not only by formal reasons ("to write a conformally invariant Lagrangian with the required properties"), but with similar physical interpretations as Englert e.a.,[17] namely "as an order parameter to indicate the spontaneous breaking of the conformal invariance" (Smolin 1979, 260). His Lagrangian used a modified adaptation from JBD theory, "with some additional couplings" between scale connection φ and scalar field ϕ. But Smolin emphasized that "these additional couplings go against the spirit of Brans-Dicke theory" as they introduced a non-vanishing divergence of the non-gravitational fields.

For low energy considerations Smolin dropped the square curvature term (in square brackets, (11)), added an "effective" potential term of the scalar field $V_{\text{eff}}(\phi)$ and derived the equations of motion by varying with respect to g, ϕ, φ. Results were Einstein equation, scalar field equation, and Yang-Mills equation for the scale connection.

Smolin's Lagrangian contained terms in the scale connection:[18]

$$-\frac{1}{4g^2}f_{\mu\nu}f^{\mu\nu} + \frac{1}{8}(1+6c)F^2\varphi_\mu\varphi^\mu \tag{12}$$

That looked like a mass term for φ considered as potential of the scale curvature field $f_{\mu\nu}$, called "Weyl field" by Smolin. By comparison with the Lagrangian of the Proca equation in electromagnetic theory, Smolin concluded that the "Weyl field" has mass close to the Planck scale, given by

$$M_\varphi^2 = \frac{1}{4}(1+6c)F^2\,. \tag{13}$$

He commented that in his Weyl geometric gravitation theory "general relativity couples to a massive vector field" φ. The scalar field ϕ, however, "may be absorbed into the scalar parts" of $g_{\mu\nu}$ and φ_μ by a change of variables and remains massless (Smolin 1979, 263). In this way, Smolin brought Weyl geometic gravity closer to the field theoretic

[15]Signs have to be taken with caution. They may depend on conventions for defining the Riemann curvature, the Ricci contraction, and the signature. Smolin, e.g., used a different sign convention for *Riem* to the one used in this survey. Signs given here are adapted to *signature* $g = (3,1)$, Riemann tensor of mathematical textbooks, and Ricci contraction like in section 2.

[16]General knowledge, made explicit, e.g. by (Hehl e.s. 1996).

[17](Englert 1975) was not quoted by Smolin.

[18]Smolin's complete Lagrangian was

$$|det\,g|^{-\frac{1}{2}}\mathcal{L}^{grav} = \frac{1}{2}c\,F^2\,{}_gR - \frac{1}{4g^2}f_{\mu\nu}f^{\mu\nu} + \frac{1}{8}(1+6c)F^2\varphi_\mu\varphi^\mu - V_{\text{eff}}(F)\,.$$

frame of particle physics. He did not discuss mass and interaction fields of the SMEP. Morover, the huge mass of the "Weyl field" must have appeared quite irritating.

Interlude. At the time Smolin's paper appeared, the program of so-called *induced gravity*, entered an active phase. Its central goal was to derive the action of conventional or modified Einstein gravity from an extended scheme of standard model type quantization. Among the authors involved in this program Stephen Adler and Anthony Zee stick out. We cannot go into this story here.[19]

Smolin's view that already the structure of Weyl geometry might be well suited to bring classical gravity into a coherent frame with standard model physics did not find much direct response. But it was "rediscovered" at least twice (plus an independently developed conformal version). In 1987/88 Hung Cheng at the MIT and a decade later Wolfgang Drechsler and Hanno Tann, both at Munich, arrived basically at similar insights. both with an explicit extension to standard model (SMEP) fields (Cheng 1988, Drechsler/Tann 1999, Drechsler 1999). Simultaneously to Cheng, the core of the idea was once more discovered by Moshé Flato (Dijon) and Ryszard Rączka (during that time at Trieste), although they formulated it in a strictly conformal framework without Weyl structure (Flato/Raçka 1988). Neither Cheng, nor Flato/Rączka or Drechsler/Tann seem to have known Smolin's proposal (at least Smolin is not cited by them); even less did they refer to the papers of each other.[20] All three approaches had their own achievements. Here we can give only give a short presentation of the main points of the work directly related to Weyl geometry.

Hung Cheng and his "vector meson". Hung Cheng started out from a Weyl geometric background, apparently inherited from the papers of Japanese authors around Utiyama. The latter had taken up Weyl geometry in the early 1970s in a way not too different from Smolin's later approach.[21] Hung Cheng extended Utiyama's theory

[19] For a survey of the status of investigations in 1981 see (Adler 1982); but note in particular (Zee 1982*b*, Zee 1983). The topic of "origin of spontaneous symmetry breaking" by radiative correction was much older, see e.g. (Coleman 1973). In fact, Zee's first publication on the subject preceded Smolin's. (Zee 1979) was submitted in December 1978 and published in February 1979; (Smolin 1979) was submitted in June 1979.

[20] Flato/Rączka's paper appeared as a preprint of the *Scuola Internazionale Superiore di Studi Avanzati*, Trieste, in 1987; the paper itself was submitted in December 1987 to *Physics Letters B* and published in July 1988. Cheng's paper was submitted in February 1988, published in November. Only a decade later, in March 2009, Drechsler and Tann got acquainted with the other two papers. This indicates that the Weyl geometric approach in field theory has not yet acquired the coherence of a research program with a stable subcommunity.

[21] (Utiyama 1973, Utiyama 1975*a*, Utiyama 1975*b*, Hayashi/Kugo 1979)

explicitly to the electroweak sector of the SMEP. The scalar field Φ of weight -1 (without a separate potential) was supposed to have values in an isospin $\frac{1}{2}$ representation.[22] Otherwise it coupled to Weyl geometric curvature R as known.

$$\mathcal{L}_R = \varepsilon \frac{1}{2} \beta\, \Phi^* \Phi R \, |det\, g|^{\frac{1}{2}} \tag{14}$$

$$\mathcal{L}_\Phi = \frac{1}{2} \tilde{D}^\mu \Phi^* \tilde{D}_\mu \Phi \, |det\, g|^{\frac{1}{2}} \, , \tag{15}$$

with $\varepsilon = 1$.[23] The scale covariant derivatives were extended to a "localized" *ew* group $SU(2) \times U(1)$. With the usual denotation of the standard model, W^j_μ for the field components of the $su(2)$ part (with respect to the Pauli matrices σ_j $(j = 0, 1, 2)$) and B_μ for $u(1)_Y \cong \mathbb{R}$ and coupling coefficients g, g' they read[24]

$$\tilde{D}_\mu \Phi = (\partial_\mu - \varphi_\mu + \frac{1}{2} i g W^j_\mu \sigma_j + \frac{1}{2} g' B_\mu) \Phi \, . \tag{16}$$

Cheng added Yang-Mills interaction Lagrangians for *ew* interaction fields F and G of the potentials W (values in su_2), respectively B (values in $u(1)_Y$), and added a scalar curvature term in $f = (f_{\mu\nu}) = d\varphi$

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} \left(f_{\mu\nu} f^{\mu\nu} + F_{\mu\nu} F^{\mu\nu} + G_{\mu\nu} G^{\mu\nu} \right) |det\, g|^{\frac{1}{2}} \, . \tag{17}$$

Finally he introduced spin $\frac{1}{2}$ fermion fields ψ with the weight convention $w(\psi) = -\frac{3}{2}$, and a Lagrangian $\mathcal{L}_\psi$ similar to the one formulated later by Drechsler, discussed below (20).[25]

Cheng called the scale connection, resp. its curvature, *Weyl's meson* field. Referring to Hayashi's e.a. observation that the scale connection does not influence the equation of motion of the spinor fields, he concluded:[26]

> ...Weyl's vector meson does not interact with leptons or quarks. Neither does it interact with other vector mesons. The only interaction the Weyl's meson has is that with the graviton. (Cheng 1988, 2183)

[22] In the sequel the isospin extended scalar field will be denoted by Φ.

[23] Drechsler and Tann would later find reasons to set $\varepsilon = -1$ (energy of the scalar field positive). Hung Cheng's curvature convention was not made explicit; so there remains a sign ambiguity.

[24] Cheng added another coupling coefficient for the scale connection, which is here suppressed.

[25] The second term in (20) is missing in Cheng's publication. That is probably not intended, but a misprint. Moreover he did not discuss scale weights for Dirac matrices in the tetrad approach.

[26] Remember that the φ terms of scale covariant derivatives in the Lagrangian of spinor fields cancel.

Because of the tremendous mass of "Weyl's vector meson" Cheng conjectured that even such a minute coupling might be of some cosmological import. More precisely, he wondered, "whether Weyl's meson may account for at least part of the dark matter of the universe" (ibid.). Similar conjectures were stated once and again over the next decades, if theoretical entities were encountered which might represent massive particles without experimental evidence. Weyl geometric field theory was not spared this experience.

Can gravity do what the Higgs does? In the same year in which Hung Cheng's paper appeared, Moshé Flato and Ryszard Rączka sketched an approach in which they put gravity into a quantum physical perspective.[27] Although it would be interesting to put this paper in perspective of point (ii) in our introduction, we cannot do it here. In our contect, this paper matters because it introduced a scale covariant Brans-Dicke like field in an isospin representation similar to Hung Cheng's, but in a strictly conformal framework (Flato/Raçka 1988).

Six years later, R. Rączka took up the thread again, now in cooperation with Marek Pawłowski. In the meantime Pawłowski had joined the research program by a paper in which he addressed the question whether perhaps gravity "can do what the Higgs does" (Pawłowski 1990). In a couple of preprints (Pawłowski/Rączka 1994*a*, Pawłowski/Rączka 1995*a*, Pawłowski/Rączka 1995*b*, Pawłowski/Rączka 1995*d*) and two refereed papers (Pawłowski/Rączka 1994*b*, Pawłowski/Rączka 1995*c*) the two physicists proposed a "Higgs free model for fundamental interactions", as they described it. This proposal is formulated in a strictly conformal setting. Although it is very interesting in itself, we cannot discuss it here in more detail.

Mass generation by coupling to gravity: Drechsler and Tann. A view close to Cheng's, establishing a connection between gravity and electroweak fields by Weyl geometry, was developed a decade later by Wolfgang Drechsler and Hanno Tann at Munich. Drechsler had been active for more than twenty years in differential geometric aspects of modern field theory.[28] Tann joined the activity during his work on his PhD thesis (Tann 1998), coming from a background interest in geometric properties of the de Broglie-Bohm interpretation of quantum mechanics. In their joint work (Drechsler/Tann 1999), as well as in their separate publications (Tann 1998, Drechsler 1999) Weyl geometric structures are used in a coherent way, clearer than in most of the other physical papers cited up to now.

[27]More than a decade earlier Flato had worked out a covariant ("curved space") generalization of the Wightman axioms (Flato/Simon 1972), obviously different from the one discussed by R. Wald in this volume, with another coauthor.

[28]For example (Drechsler 1977).

They arrived, each one on his own, at the full expression for the (metrical) energy momentum tensor of the scalar field, including terms which resulted from varying the scale invariant Hilbert-Einstein term (containing the factor ξ^{-1}).[29]

$$(18)\quad T_\phi = D_{(\mu}\phi^* D_{\nu)}\phi - \xi^{-1} D_{(\mu} D_{\nu)} |\phi|^2 - g_{\mu\nu}\left(\frac{1}{2} D^\lambda \phi^* D_\lambda \phi - \xi^{-1} D^\lambda D_\lambda(\phi^*\phi) + V(\phi)\right) .$$

In their common paper, Drechsler and Tann introduced fermionic Dirac fields into the analysis of Weyl geometry (Drechsler/Tann 1999). Their gravitational Lagrangian had the form

$$\mathcal{L}_{\text{grav}} = \mathcal{L}_R + \mathcal{L}_{R^2} \tag{19}$$

with $\mathcal{L}_R$ identical to Hung Cheng's (14), in addition to $\mathcal{L}_\phi$ (15) (with coefficients $\beta = \frac{1}{6}$, $\varepsilon = -1$). A quadratic term, $\mathcal{L}_{R^2} = \tilde{\alpha} R^2 \sqrt{|det\, g|}$, in the (Weyl geometric) scalar curvature, was added.[30]

For the development of a Weyl geometric theory of the Dirac field, Drechsler and Tann introduced an adapted Lagrangian

$$\mathcal{L}_\psi = \frac{i}{2}\left(\psi^* \gamma^\mu D_\mu \psi - D^*_\mu \psi^* \gamma^\mu \psi\right) + \gamma |\Phi| \psi^* \psi \tag{20}$$

with (scale invariant) coupling constant γ and Dirac matrices γ^μ with symmetric product $\frac{1}{2}\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu}\mathbf{1}$ (Drechsler/Tann 1999, (3.8)). Here the covariant derivative had to be lifted to the spinor bundle, It included already an additional $U(1)$ electromagnetic potential $A = (A_\mu)$

$$D_\mu \psi = \left(\partial_\mu + i\tilde{\Gamma}_\mu + w(\psi)\varphi_\mu + \frac{iq}{\hbar c} A_\mu\right)\psi \ , \tag{21}$$

q electric charge of the fermion field, $w(\psi) = -\frac{3}{2}$, $\tilde{\Gamma}$ spin connection lifted from the Weylian affine connection. This amounted to a (local) construction of a spin $\frac{1}{2}$ bunde. Assuming the underlying spacetime M to be spin, they worked in a Dirac spin bundle $\mathcal{D}$ over the Weylian manifold $(M, [(g, \varphi)]$. Its structure group was $G = Spin(3,1) \times R^+ \times U(1) \cong Spin(3,1) \times \mathbb{C}^*$, where $\mathbb{C}^* = \mathbb{C} \setminus 0$.[31]

---

[29] The terms with factor ξ^{-1} had been introduced in an "improved energy-momentum tensor" by Callan (1970) in a more ad-hoc way; cf. (Tann 1998, (372)).

[30] In the appendix Drechsler and Tann showed that the squared Weyl geometric conformal curvature $C^2 = C_{\lambda\mu\nu\rho} C^{\lambda\mu\nu\rho}$ arises from the conformal cuvature of the Riemannian component ${}_gC^2$ by adding a scale curvature term: $C^2 = {}_gC^2 + \frac{3}{2} f_{\mu\nu} f^{\mu\nu}$ (Drechsler/Tann 1999, (A 54)). So one may wonder, why they did not replace the square term $\mathcal{L}_{R^2}$ by the Weyl geometric conformal curvature term $\mathcal{L}_{\text{conf}} = \tilde{\alpha} C^2 \sqrt{|det\, g|}$.

[31] One could then just as well consider a complex valued connection $z = (z_\mu)$ with values $z_\mu = \varphi_\mu + \frac{i}{\hbar c} A_\mu$ in $\mathbb{C} = Lie(\mathbb{C}^*)$ and weight $W(\psi) = (-\frac{3}{2}, q)$. Then $D_\mu \psi = (\partial_\mu + \tilde{\Gamma}_\mu + W(\psi) z_\mu)\psi$, presupposing an obvious convention for applying $W(\psi) z$.

Drechsler and Tann considered (20) as Lagrangian of a "massless" theory, because the masslike factor of the spinor field $\gamma|\Phi|$ was not scale invariant.[32] So they proposed to proceed to a theory with masses by introducing a "scale symmetry breaking" Lagrange term

$$\mathcal{L}_B \sim \frac{R}{6} + (\frac{mc^2}{\hbar})^2 |\Phi|^2 \tag{22}$$

with fixed (non-scaling) m (Drechsler/Tann 1999, sec. 4).[33]

They did not associate such a transition from a seemingly "massless" theory to one with masses to a hypothetical "phase transition". At the end of the paper they commented:

> It is clear from the role the modulus of the scalar field plays in this theory (...) that the scalar field with nonlinear selfcoupling is not a true matter field describing scalar particles. It is a universal field necessary to establish a scale of length in a theory and should probably not be interpreted as a field having a particle interpretation. (Drechsler/Tann 1999, 1050)

Their interpretation of the scalar field Φ was rather geometric than that of an ordinary quantum field; but their term (22) looked ad-hoc to the uninitiated.[34]

Drechsler on mass acquirement of electroweak bosons. Shortly after the common article appeared, the senior author extended the investigation to gravitationally coupled electroweak theory (Drechsler 1999). Covariant derivatives were lifted as $\tilde{D}$ to the electroweak bundle. It included the additional connection components and coupling coefficients g and g' with regard to $SU(2)$ and $U(1)_Y$ like in Hung Cheng's work (16). The Weyl geometric Lagrangian could be generalized and transferred to the electroweak bundle (Drechsler 1999, (2.29)),

$$\mathcal{L} = \mathcal{L}_{\text{grav}} + \mathcal{L}_{\Phi} + \mathcal{L}_{\psi} + \mathcal{L}_{\text{YM}}\,, \tag{23}$$

with contributions like in (19), (15), (20), and (17) (*ew* terms only). Lagrangians for the fermion fields had to be rewritten similar to electromagnetic Dirac fields (20) and were decomposed into the chiral left and right contributions.

In principle, Drechsler's proposal coincided with Cheng's; but he proceeded with more care and with more detailed explicit constructions. He derived the equations of motion with respect to all dynamical variables (Drechsler 1999, equs. (2.35) – (2.41)) and calculated the energy-momentum tensors of all fields ocurring in the Lagrangian.

[32]This argument is possible, but not compelling. $\gamma|\Phi|$ has the correct scaling weight of mass and may be considered as such.

[33] So already in Tann's PhD dissertation.

[34]Note that one could just as well do without (22) and proceed with fully scale covariant masses – compare last footnote.

The symmetry reduction from the electroweak group G_{ew} to the electromagnetic $U(1)_{\text{em}}$ could then be expressed similar to the procedure in the standard model. $SU(2)$ gauge freedom allows to chose a (local) trivialization of the electroweak bundle such that the Φ assumes the form considered in the ordinary Higgs mechanism

$$\hat{\Phi} \doteq \begin{pmatrix} 0 \\ \phi_o \end{pmatrix} , \tag{24}$$

where ϕ_o denotes a real valued field, and "$\doteq$" equality in a specific gauge. $\hat{\Phi}$ has the isotropy group $U(1)$ considered as $U(1)_{\text{em}}$. Therefore Drechsler called $\hat{\Phi}$ the *electromagnetic gauge* of Φ.[35]

In two respects Drechsler went beyond what had been done before:

- He *reconsidered the standard interpretation* of symmetry breaking by the Higgs mechanism (Drechsler 1999, 1345f.).
- And he *calculated* the consequences of nonvanishing electroweak curvature components for the *energy-momentum tensor* of the scalar field $\hat{\Phi}$ (Drechsler 1999, 1353ff.).

With regard to the first point, he made clear that he saw nothing compelling in the interpretation of symmetry reduction as "spontaneous symmetry breaking due to a nonvanishing vacuum expectation value of the scalar field" (Drechsler 1999, 1345). He analyzed the situation and came to the conclusion that the transition from our Φ to $\hat{\Phi}$ is to be regarded as a "choice of coordinates" for the representation of the scalar field in the theory and has, in the first place, nothing to do with a "vacuum expectation value" of this field.[36]

> ... This choice is actually not a breaking of the orginal $\tilde{G}$ gauge symmetry [our G_{ew}, E.S.] but a different realization of it. (ibid.)

He compared the stabilizer $U(1)_{\text{em}}$ of $\hat{\Phi}$ with the "Wigner rotations" in the study of the representations of the Poincaré group. With regard to the second point, the energy-momentum tensor of the scalar field could be calculated roughly like in the simpler case of a complex scalar field, (18). Different to what one knew from the pseudo-Riemannian case, the covariant derivatives $D_\mu \Phi$ etc. in (18) were then dependent on scale or $U(1)_{\text{em}}$ curvature.

After breaking the Weyl symmetry by a Lagrangian of form (22) (ibid. sec. 3), Drechsler calculated the curvature contributions induced by the Yang-Mills potentials of the *ew* group and its consequences for

[35] In other parts of the literature (e.g., the work of Rączka and Pawłowski) it is called "unitary gauge", cf. also (Flato/Raçka 1988).

[36] Mathematically spoken, it is a change of trivialization of the $SU(2) \times U(1)$-bundle.

the energy-momentum tensor T_ϕ of the scalar field. Typical contributions to components of T_ϕ had the form of mass terms

(25)
$$m_W^2 W_\mu^{+\,*} W^{-\,\mu},\ m_Z Z_\mu^* Z^\mu\,, \quad \text{with} \quad m_W^2 = \frac{1}{4} g^2 |\phi_o|^2,\ m_Z^2 = \frac{1}{4} g_o^2 |\phi_o|^2\,,$$

$g_o^2 = g^2 + g'^2$, for the bosonic fields $W^\pm, Z$ corresponding to the generators $\tau_\pm, \tau_o$ of the electroweak group, (Drechsler 1999, 1353ff.).[37] They are identical with the mass expressions for the W and Z bosons in conventional electroweak theory. According to Drechsler, the terms (25) in T_ϕ indicate that the" boson and fermion mass terms appear in the total energy-momentum tensor" through the energy tensor of the scalar field after "breaking the Weyl symmetry".[38] Inasmuch as the scalar field can be considered as extension of the gravitational structure of spacetime, the scale covariant theory of mass acquirement indicates a way to *mass generation* by coupling to the gravitational structure. In any case, one has to keep in mind that the scalar field "...should probably not be interpreted as a field having a particle interpretation" (Drechsler/Tann 1999, 1050).

Such a type of mass generation would have remarkable observable consequences in the LHC regime. The decay channels involving the standard Higgs particle would be completely suppressed.[39] If the LHC experiments turn out as a giant null-experiment with regard to chasing the Higgs particle, the scale covariant scalar field should run up as a serious theoretical alternative to the Higgs mechanism.

4. Scale covariance in recent cosmology

Recent uses of Weyl geometry in cosmology. Already early in the 1990s Rosen and Israelit studied different possibilities for "generating" dark matter in Dirac's modified Weyl geometric framework (Israelit/Rosen 1992, Israelit/Rosen 1993, Israelit/Rosen 1995, Israelit/Rosen 1996). They presupposed a non-integrable scale connection

[37] $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$, $Z_\mu = \cos\Theta\, W_\mu^3 - \sin\Theta\, B_\mu$.

[38] One has to be careful, however. Things become more complicated if one considers the trace. In fact, $tr\, T_\phi$ contains a mass terms of the Dirac field of form $\gamma|\phi_o|\hat\psi^*\hat\psi$, with γ coupling constant of the Yukawa term ($\hat\psi$ indicating electromagnetic gauge). That should be interesting for workers in the field. One of the obstacles for making quantum matter fields compatible with classical gravity is the vanishing of $tr\, T_\psi$, in contrast to the (nonvanishing) trace of the energy momentum tensor of classical matter. Drechsler's analysis may indicate a way out of this impasse. Warning: The mass-like expressions for W and Z in (25) cancel in $tr\, T_\phi$ (Drechsler 1999, equ. (3.55)) like in the energy-momentum tensor of the W and Z fields themselves. In this sense, the mass terms of fermions and those of electroweak bosons behave differently with regard to the energy momentum tensor T_ϕ.

[39] A calculation of radiative corrections in the closely related conformal approach is presented in (Pawłowski/Rączka 1995*d*); comparison with (Kniehl/Sirlin 2000) might be informative for experts.

leading to a spin 1 boson field which satisfied a scale covariant Proca equation, like in Utiyamah's, Smolin's and Hung Cheng's papers. The authors called the new hypothetical bosons *Weylons* and proposed a crucial role for them in the constitution of dark matter. In recent years M. Israelit has developed ideas, how matter may even have been "generated from geometry" in the very early universe (Israelit 2002*a*)[40] and added a "quintessence" model in the framework of the Weyl-Dirac geometry (Israelit 2002*b*). Not all of it is convincing; but here is not the place to go into details.

Weyl geometry has been reconsidered also by other authors as a possibility to relax the structural restrictions of Einsteinian gravity in a natural and, in a sense, minimal way. That happened indepently at several places in the world, at Tehran, Beijing, Santa Clara, Wuppertal, Atlanta, and perhaps elsewhere. Some of these attempts built upon the Rosen/Israelit tradition of Weyl-Dirac geometry, others linked to the field theoretic usage of Weyl geometry in the standard model of elementary particle physics, or to the Weyl geometric interpretation of the de Broglie-Bohm quantum potential.

Entering the new millenium, our selective report will definitely leave the historical terrain in the proper sense. By pure convention I consider work after the watershed of the year 2000 as "present". It may, or may not, become object of historical research in some more or less distant future. The remaining section concentrates on those contributions of scalar fields or scale covariant aspects in cosmology which relate directly or indirectly to more basic aspects of Weyl geometry.

The scalar-tensor approach to gravity in the sense of Brans-Dicke theory has been studied all over the world. Among those active in this field, Israel Quiros from the university at Santa Clara (Cuba) realized that the "transformations of units" in the sense of Brans-Dicke finds its most consequent expression in Weyl geometry. In some papers around the turn of the millenium he argued in this sense (Quiros 2000*a*, Quiros 2000*b*); but his main work remains in more mainstream field theory and cosmology.

A turn of longer endurance toward Weyl geometry was taken by M. Golshani, F. and A. Shojai, from the Tehran theoretical physics community. Their interest stabilized when they studied the link between Brans- Dicke type scalar tensor theory and de Broglie-Bohm quantum mechanics. About 2003 (perhaps during their stay at the MPI for gravitation research Golm/Potsdam) the Shojais realized that Weyl geometry can be used as a a unifying frame for such an enterprise (Shojai/Shojai 2002). It seems that their retake of Weyl geometry may have influenced other colleagues of the local physics community, who started to analyze astrophysical questions by Weyl geometric methods (Moyassari/Jalalzadeh 2004, Mirabotalebi e.a. 2008). The Weyl

[40]Compare with H. Fahr's proposal in his contribution to this volume.

geometric background knowledge of the Tehran group was shaped by Weyl-Dirac theory and the Rosen/Israelit tradition, supplemented by the analysis of Ehlers/Pirani/Schild (1972) and of Wheeler (1990). The latter had explored the relation between quantum physics and (Weyl) geometry already back in the 1990s.

E. Scholz, a historian of mathematics at the Mathematics Department of Wuppertal University (Germany) started studying Weyl geometry in present cosmology shortly after attending a conference on history of geometry at Paris in September 2001.[41] Coming from a background in mathematics and its history, it took some time before he got acquainted with the more recent Weyl geometric tradition in theoretical physics. After he "detected" the work of Drechsler and Tann on Weyl geometric methods in field theory in late 2004, it became a clue for his entering the physics discourse in field physics (Scholz 2005*b*, Scholz 2009).

C. Castro had become acquainted with Weyl geometry in physics already in the early 1990s while being at Austin/Texas. At that time a proposal by E. Santamato's to use Weyl geometry for a geometrization of de Broglie-Bohm quantum mechanics stood at the center of his interest (Castro 1992).[42] After the turn of the millenium, then working at Atlanta (Centre for Theoretical Studies of Physical Systems), he took up the Weylian thread again, now with the guiding questions, how Weyl's scale geometry may be used for gaining a deeper understanding of dark energy and, perhaps, the Pioneer anomaly (Castro 2007, Castro 2009).

A completely different road towards Weyl geometry was opened for Chinese theoretical physicists Hao Wei, Rong-Gen Cai, and others by a talk of Hung Cheng, given in July 2004 at the Institute for Theoretical Physics of the Chinese Academy of Science, Beijing.[43] It was natural for them to take the "'Cheng-Weyl vector field" (i.e., the Weylian scale connection with massive boson studied by Cheng in the late 1980s) and Cheng's view of the standard model of elementary particle physics as their starting point (Wu 2004, Wei 2007).

So far only groups or persons have been mentioned, who contributed explicitly to the present revitalization of Weylian scale geometry. Other protagonists whose work plays a role for this question will enter this section, even if they do not care about links to Weyl geometry.

---

[41]On this conference P. Cartier, a protagonist of the second generation of the Bourbaki group who has been interested in mathematical physics all his life, gave an enthusiastic talk on the importance of Weyl's scale connection for understanding cosmological redshift (Cartier 2001). Scholz was struck by this talk, because he had tried to win over physicists for such an idea in the early 1990s, of course without any success.

[42](Santamato 1985, Santamato 1984)

[43](Wei 2007, Acknowledgments)

Mannheim's conformal cosmology. A striking analysis of certain aspects of recent cosmology (the *dark* ones, dark matter and dark energy) was given by Philip Mannheim and Demosthenes Kazanas. In the 1980s the two physicists analyzed the "flat" shape of galaxy rotation curves (graphs of the rotation velocity v of stars in dependence of the distance r from the center of the galaxy). From a certain distance close to a characteristic length of the galaxy ($2.2\, r_o$ with r_o the "optical disc length scale") v is greater than expected by Newtonian mechanics, such that the spiral should have flewn apart unless unseen ("dark") matter enhancing gravitational binding or a modification of Newtonian/Schwarzschild gravity were assumed. While the majority of astrophysicists and astronomers assumed the first hypothesis, Mannheim looked for possible explanations along the second line. In (Mannheim 1989) a theoretical explanation of the flat rotation curves was given, based on a conformal approach to gravity. During the following years the approach was deepened and extended to the question of "dark energy".

In fact Mannheim and Kazanas found that, in the conformal theory, a static spherically symmetric matter distribution could be described by the solution of a fourth order Poisson equation

$$\nabla^4 B(r) = f(r) \tag{26}$$

with a typical coefficient $B(r)$

$$B \sim -g_{oo} = g_{rr}^{-1}$$

of a metric $ds^2 = g_{oo}dt^2 - g_{rr}dr^2 - r^2 d\Omega^2$ (up to a conformal factor). The r.h.s. of the Poisson equation, $f(r)$, depended on the mass distribution, e.g., in a spiral galaxy.

A general solution turned out to be of the form

$$g_{oo} = 1 - \beta(2 - 3\beta\gamma)r^{-1} - 3\beta\gamma + \gamma r - \kappa r^2 \tag{27}$$

with constants β, γ, κ. Here β depends on the mass and its distribution in the galaxy. For galaxies Mannheim arrived at such small values for β and γ that the $\beta\gamma$ terms could be numerically neglected, as could the r^2 term. In this case the β term took on the form of a Schwarzschild solution of the Einstein equation with Schwarzschild radius $r_S = \beta$. The classical potential was, however, modified by a term linear in r, in addition to the classical Newton potential,

$$V(r) = -\frac{\beta}{r} + \frac{\gamma}{2} r \tag{28}$$

and a corresponding velocity of generalized Kepler orbits

$$v(r) \sim V(r)\,. \tag{29}$$

The dynamics of such a potential agreed well with the data of galactic rotation curves (Mannheim 1989, Mannheim 1994).

The result for 11 galaxies with different behaviour of rotation curves led to surprisingly good fit. γ was basically independent of the galaxy. It had a cosmological order of magnitude, $\gamma \approx 3 \cdot 10^{-30}\, cm^{-1} \approx 0.04 H_1$. Mannheim considered this “an intriguing fact which suggests a possible cosmological origin for γ” (Mannheim 1994, 498). In his view, it represented a kind of (weak) Machian type influence of very distant masses on the potential, which could be neglected close to stars and at the center of the bulge of galaxies. It came to bear only at the galactic periphery and beyond.

With respect to the dark energy problematics, Mannheim chose a peculiar perspective. In the special case of conformally flat models, like Robertson-Walker geometries, he decided to consider the scale invariant Hilbert-Einstein Lagrangian $-\frac{1}{12}|\phi|^2 R\sqrt{|det\, g|}$ as part of the *matter* Lagrangian. Due to his sign choices, he arrived at a version of the Einstein equation with *inverted* sign and interpreted it as a kind of “repulsive gravity” which he claims to operate on cosmic scales, in addition to “attractive gravity” on smaller scales indicated by the conformally modified Schwarzschild solution. In his eyes, such a repulsive gravity might step into the place of the dark energy of the cosmological constant term of standard gravity (Mannheim 2000, 729).

In spite of such a grave difference to Einstein gravity, Mannheim does not consider his conformal approach to disagree with the standard model of cosmology and its accelerated expansion. He rather believes that his approach may lead to a more satisfying explanation of the expansion dynamics. In his view, “repulsive gravity” would take over the role of dark energy. Moreover he expects that it may shed new light on the initial singularity and, perhaps, also on the black hole singularities inside galaxies.

Scalar fields and dark energy: Kim and Castro. Other authors started to analyze dark energy by a scalar field approach. A remarkable contribution to this topic comes from Hongsu Kim (Seoul). He uses a classical Jordan-Brans-Dicke field, $\chi = |\phi|^2$ with JBD parameter ω like in (9) and shows that, under certain assumptions, it may lead to a phase of *linear expansion* in “late time” development of the cosmos, i.e. long after inflation but long before “today” (Kim 2005) . He proposes to consider a transitional phase between a matter dominated phase with decelerated expansion (decelerated because of dominance of gravitational attraction over the repulsive vacuum energy) and an accelerated expansion dominated by vacuum energy. For the JBD field he uses the funny terminology of *k-essence*, in distinction to *quintessence* which has been introduced for scalar fields without coupling to gravity.

Assuming a spatially flat Robertson-Walker spacetime with warp (scale) function $a(t)$, without ordinary matter and without cosmological constant, Kim starts from the modified Friedmann and scalar field equations:

$$\begin{aligned} \left(\frac{\dot a}{a}\right)^2 &= \frac{\omega}{6}\left(\frac{\dot\chi}{\chi}\right)^2 - \frac{\dot a}{a}\left(\frac{\dot\chi}{\chi}\right) \\ \ddot\chi + 3\frac{\dot a}{a}\chi &= 0 \end{aligned}$$

(t time parameter, χ scalar field, ω JBD parameter).

By an *Ansatz* of the form $a(t) \sim (at+b)^\alpha,\ \chi(t) \sim a(t)^n$ he evaluates the energy stress tensor of χ. The divergence condition $\nabla^\nu T^{(\chi)}_{\mu\nu} = 0$ implies the restriction $\omega = -\frac{3}{2}\ \ n = -2$. This leads to a solution with linear warp function $a(t) \sim t$, and $\rho_\chi \sim a(t)^{-2}\,, \qquad p_\chi = -\frac{1}{3}\rho_\chi$ for the energy density ρ_χ and pressure p_χ of the scalar field. According to the author, this solution indicates a kind of dynamically neutral intermediate state of the underlying universe model between deceleration and acceleration, which does not arise in the standard model (Kim 2005, equs. (8), (13), (19)).

Kim goes on to consider a Lagrangean with JBD field term (9), including ordinary matter m and cosmological constant Λ. He assumes that the "late" time evolution of the cosmos consists of three phases:

- at the beginning of "late time" matter dominates the other two terms (deceleration),
- in an intermediate stage matter density has been diluted sufficiently far so that the scalar field dominates the dynamics (linear expansion, m and Λ negligible),
- after further dilution of ρ_χ, the vacuum term Λ takes over and dominates the evolution of the model (acceleration).

In the first phase χ "mixes" with (ordinary) matter. Kim considers this as a state of dark matter. In the third phase χ seems to mix with the vacuum term. The scalar field is then assimilated to dark energy. In this sense, so Kim argues, JBD theory offers "a unified model" for dark matter and dark energy.

Kim's analysis of the contribution of JBD fields to cosmological dynamics is highly interesting; but it has two empirical drawbacks. Firstly and noticed by himself, the present standard model does not know of any linear phase of expansion.[44] Much worse, although not discussed by Kim, is the empirical restriction for the JBD parameter $\omega > 10^3$, following from the comparison of post-Newtonian parametrized gravity

[44]This is no great handicap because the paradigm of standard cosmology does not allow such a phase and therefore could not "see" it, even if it existed. Only extreme observational results could *enforce* such a phase onto the standard view and would then break it up.

theories with the data of high precision observations.[45] This observational result stands in grave contradiction to Kim's theoretical value $\omega = -\frac{3}{2}$.[46]

Some of our authors assume

$$D_\mu \phi = 0 \ , \tag{30}$$

for the sake of simplicity (Kao 1990), others by their background in de Broglie-Bohm theory (Santamato 1984, equ. (14)). In the context of scalar tensor theory, however, this condition introduces a dynamical overdetermination and makes Weyl geometry essentially redundant. This can be seen by comparison with the JBD approach. In Riemann gauge, condition (30) implies $\partial_\mu \phi = 0$ and thus $|\phi| = const.$ In terms of of Brans-Dicke theory one arrives at a constant scalar field in Jordan frame (something like a *contradictio in adjecto*)! Then the latter is identical with the Einstein frame, and the whole JBD theory becomes trivial. In the sequel I shall call assumption of (30) a *trivial Weylianization* of Einstein gravity.

Castro arrives at condition (30) by varying the Weyl geometric Lagrangian (14), (15)

$$\mathcal{L} = \mathcal{L}_R + \mathcal{L}_\Phi + \mathcal{L}_m$$

not only with respect to g, ϕ and the matter variables, but also with respect to φ (Castro 2007, equ. (3.21) ff.), (Castro 2009, equ. (10)). As we have just seen, φ is no independent dynamical variable in the scalar field approach to *integrable* Weyl geometry. It is nothing but the "other side" of the scalar field which has a dynamical (Klein- Gordon) equation of its own.[47] With some additional assumptions on a de Sitter solution he "derives" a constant vacuum energy of the "right order of magnitude".

---

[45]Cf. the contribution of C. Will, this volume.

[46]One should keep in mind, however, that the observational constraint for ω refers to *Jordan frame*, which Kim presupposes at the moment. In the Weyl geometric reading of JBD theory the Einstein frame would appear more appropriate for observational purposes (see below, under equ. (33)).

[47]In order to avoid misunderstanding, it has to be added that φ is, of course, a dynamical variable in the nonintegrable case. Then one has to introduce a scale curvature term into the Lagrangian (usually of Yang-Mills type, $\frac{1}{4} f_{\mu\nu} f^{\mu\nu}$ like in the work of Dirac, Smolin (11), Hung Cheng and Drechsler/Tann. The integrable case arises from constraining conditions which can be expressed in the action by a system of (antisymmetric) Lagrange multipliers $\lambda^{\mu\nu}$ for scale curvature $f = (f_{\mu\nu})$,

$$\mathcal{L}_f = \left(\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{1}{2} \lambda^{\mu\nu} f_{\mu\nu} \right) \sqrt{|det\, g|} \ . \tag{31}$$

The variation considered by Castro then acquires an additional term from the derivatives of the Lagrange multipliers, and the result looks something like $D_\mu \phi = D_\nu \lambda^\nu_\mu$. Variation with respect to the multipliers $\lambda^{\mu\nu}$ gives the integrability constraint $f_{\mu\nu} = 0$.

Taking rescaling seriously: Scholz, Masreliez. In JBD theory the physical consequences of conformal rescaling have long been discussed, with a tendency toward shifting from Jordan frame to Einstein frame as the preferred one for physical observations (Faraoni e.a. 1998). A similar discussion in Weyl geometry is nearly lacking. One of the exceptions is a passage in (Scholz 2009). This paper also presents a quite different direction for a Weyl geometric analysis of questions relating to dark energy. Scholz, starts from Drechsler's and Tann's work but simplifies the Lagrangian for a classical approch to cosmology.

He abstracts from the particle fields of the SMEP and plugs in a classical matter term of an ideal fluid defined with respect to a timelike unit vector field $X = (X^\mu)$ like in (Hawking/Ellis 1973, 69f). These have to be adapted to the Weyl geometric approach by ascribing proper scale weights, $w(\rho_m) = -4$ and $w(X) = -1$, to matter energy density ρ_m and to the unit field X.[48] From the results of Ehlers/Pirani/Schild and Audretsch/Gähler/Straumann Scholz draws the consequence of postulating integrability of the Weylian scale connection, $d\varphi = 0$.[49] Abstracting from radiation, in a first approach, he arrives at a Weyl geometric (scale invariant) version of the Einstein equation,

$$Ric - \frac{1}{2}Rg = (\xi|\phi|)^{-2}(T^{(m)} + T^{(\phi)}) \tag{32}$$

with matter term $T^{(m)}_{\mu\nu} = (\rho_m + pm)X_\mu X_\nu + p_m g_{\mu\nu}$ well known from ideal fluids, but here appearing with scale covariant matter energy density and pressure p_m. The energy stress tensor of the scalar field $T^{(\phi)}$ has been computed by Drechsler/Tann (18). ξ is a coefficient regulating the relative coupling strengths of ϕ to the scale invariant Hilbert- Einstein term in comparison to its kinetic term, like in (10). In this paper Scholz follows the prescription from conformal gravity that ξ should be set to $\frac{1}{6}$ for spacetime dimension $n = 4$, although in the Weyl geometric frame scale invariance of the Lagrangian holds for any value of ξ; and ξ is an adaptable parameter, at least at the outset.[50]

Close to Weyl's "calibration by adaptation" Scholz introduces "scale invariant magnitudes" $\hat{Y}$ of any scale covariant quantity Y (scalar, vectorial, tensorial etc.) at a point p by[51]

$$\hat{Y}(p) = |\phi(p)|^w \, Y(p) \, , \qquad w := w(Y) \, . \tag{33}$$

[48] Such an assumption is completely natural from "transformation of units" view of scale transformations.

[49] (Ehlers/Pirani/Schild 1972, Audretsch/Gähler/Straumann 1984)

[50] The Weyl geometric Einstein-Hilbert Lagrangian $\xi|\phi|^2 R\sqrt{|det\, g|}$ and the kinetic term $D^\mu\phi^* D_\mu\phi\sqrt{|det\, g|}$ are indepently scale invariant. In the conformal theory they are scale invariant only as a joint package after the choice of $\xi = \frac{n-2}{4(n-1)}$.

[51] Cf. footnote 48.

Observable quantities can be evaluated in any scale gauge; $\hat{Y}$ is gauge invariantly defined. In this sense all gauges have equal status. But the determination is easiest in scalar field gauge with $|\phi| = const$, corresponding to "Einstein frame" in the terminology of JBD theory, because in this gauge $\hat{Y}(p) \doteq Y(p)$.[52]

Robertson-Walker cosmologies arise here from the usual assumption of a global foliation with homogenous and isotropic spacelike folia orthogonal to the timelike vector field X, which is now identified with an observer field specified by the flow.

Scholz hints at the striking property that cosmological redshift z of a signal emitted at a point p_1 and observed at p_2 (with respect to the observer field) after following a null geodesic γ is scale invariant,[53]

$$z + 1 = \frac{g_{p_1}(\gamma'(p_1), X(p_1))}{g_{p_2}(\gamma'(p_2), X(p_2))} \,. \tag{34}$$

Thus the Weyl geometric look at Brans-Dicke fields shows two things: Firstly, the rescaling to scalar field gauge (Einstein frame) is "natural" in the sense of giving direct access to observable magnitudes, although not the only one in which observable magnitudes can be calculated. Second, cosmological redshift is not exclusively bound to "expansion" (warping by $a(t)$); it may just as well depend on the scale connection or the scalar field. In $|\phi|$-gauge it is an effect composed by a warp function contribution and by Weyl's scale transfer function

$$\lambda(p_1, p_2) = e^{\int_1^2 \varphi(\gamma')} \tag{35}$$

(γ path between points 1 and 2).

In certain cases the residual expansion vanishes and cosmological redshift is exclusively given by the scale transfer, $z + 1 = \lambda$. In this case z can just as well be attributed to the scalar field (in Riemann gauge/Jordan frame) as to the Weylian scale connection linked to it (in scalar field gauge/Einstein frame). *Expansion no longer appears as a physically real effect of cosmology.* The warp function $a(t)$ is reduced to an auxiliary role in the mathematics of Robertson-Walker-Weyl geometries.

The final aim of Scholz' investigation is a study of those Robertson-Walker-Weyl geometries in which such an extreme reduction of cosmological redshift to the scale connection (respectively the scalar field) happens. In honour of the inventor of the mathematical framework used by him he calls them "Weyl universes". Summarily stated a *Weyl universe* is given, in scalar field gauge, by a static spacetime geometry with Riemannian component g of the Weylian metric with spatial

[52]Cf. Utiyama's and Israelit's view of ϕ as a "measure field".

[53]The invariance of z is due to the natural scaling of the timelike field, the invariance of (null and other) geodesics and the scaling of the metric.

slices of constant sectional curvature $\kappa = ka^{-2}$, $k = 0, \pm 1$ (a "radius of curvature"),

$$ds^2 = -d\tau^2 + \frac{dr^2}{1-\kappa\, r^2} + r^2(d\Theta^2 + r^2 \sin^2\Theta\, d\phi^2)\,. \tag{36}$$

The scale connection is time homogeneous with only nonvanishing component H (constant) in time direction,

$$\varphi = (H, 0, 0, 0)\,. \tag{37}$$

The scalar field is, of course, also constant,

$$|\phi|^2 = \xi^{-1}\frac{c^4}{8\pi G} = 6\frac{c^4}{8\pi G}\,. \tag{38}$$

For the transition to Riemann gauge $\tilde{g} = \Omega^2 g$, $\tilde{\varphi} = 0$ (Jordan frame) the Weylian scale transfer function is used, $\Omega(t) = e^{Ht}$. That leads to the metric,

$$d\tilde{s}^2 = e^{2Ht}\left(-d\tau^2 + \frac{dr^2}{1-\kappa\, r^2} + r^2(d\Theta^2 + r^2\sin^2\Theta\, d\phi^2)\right)\,, \tag{39}$$

called *scale expanding cosmos* by J. Masreliez (see below), and to an exponentially decaying scalar field $\tilde{\phi}(t) = e^{-Ht}$.

A change of time coordinate, $\tau = e^{Ht}$ shows that the "scale expanding cosmos" (39) is nothing but a *linearly expanding Robertson-Walker-Weyl model.* It has an inversely decaying scalar field which influences observable quantities in the sense of (33),

$$d\tilde{s}^2 = -d\tau^2 + (H\tau)^2 a^2\left(\frac{dr^2}{1-k\, r^2} + r^2(d\Theta^2 + r^2\sin^2\Theta\, d\phi^2)\right), \tag{40}$$

$$\tilde{\phi}(\tau) = (H\tau)^{-1}\,, \qquad \tilde{\varphi} = 0\,. \tag{41}$$

The observable Hubble parameter is $\hat{H} = H$.[54]

Close to the end of (Scholz 2009) the author investigates conditions under which the energy stress tensor of the scalar field stabilizes an Einstein-Weyl universe (i.e. one with positive sectional curvature $\kappa > 0$). He comes to the conclusion that this may happen if a a certain relation between H, mass energy density ρ_m and the coefficient λ_4 of the fourth order term of the scalar field potential ($V(\phi) = \lambda_4|\phi|^4$) is satisfied (Scholz 2009, 64). This condition seems neither particularly natural nor theoretically impossible. Although one may reasonably doubt that this is the end of the story, it demonstrates the existence of unexpected possibilites for the energy stress tensor of the scalar field

[54]The Hubble coefficient $\tilde{H}$ is measured as a (reciprocal) energy reduction of photons over distance (resp. running time) and has dimension inverse length. It is a magnitude of scale weight $w(\tilde{H}) = -1$. The Hubble parameter of ordinary Robertson-Walker theory is $H(\tau) = \frac{\dot{a}}{a} = \tau^{-1}$. Its Weyl geometric observable magnitude (33) is thus $\hat{H}(\tau) = |\phi(\tau)|^{-1}H(\tau) = H$.

in the Weyl geometric approach. In fact, they are excluded in Einstein gravity by the singularity theorems of Penrose and Hawkins.[55]

Conceptually Weyl universes are closely connected to the theory of *scale expanding cosmos* (SEC), proposed by J. Masreliez (Masreliez 2004*a*, Masreliez 2004*b*). Masreliez works with a metric like in (39) and tries to rebuild more or less the whole of cosmology. He doubts the reality of cosmic expansion from a physicists point of view[56] and argues for "physical equivalence" of the scale expanding metric very much in the sense of scale co/invariant theory:

> [S]cale expansion for flat or curved spacetimes does not alter physical relationships; scaled spacetimes are equivalent and scale invariance is a fundamental, universal, gauge invariance. (Masreliez 2004*a*, 104)

Masreliez calls upon scale invariant theory and, in the end, of Weyl geometry. His SEC model is basically nothing but a Weyl universe, considered in Riemann gauge. He generally prefers a flat SEC, or in our terms, a Minkowski-Weyl universe. But he does not realize that Weyl geometry might be helpful for his enterprise. We cannot enter into details here, because not all of Masreliez' explanations are mature; many seem not particularly clear to the reporting author.

Attempts for understanding "dark matter". Besides the more widely noticed approach of modified Newtonian dynamics (MOND) and Mannheim's conformal gravitation, some researchers try to understand the flat rotation curves of galaxies by the contribution of scalar fields to the total energy around galaxies and clusters. Franz Schunck started with such research already in his Cologne PhD thesis under the direction of E. Mielke (Schunck 1995, Schunck 1999). He continued this line of investigation in different constellations. Here we concentrate on joint work with Mielke and Burkhard Fuchs (Fuchs e.a. 2006).[57] Already in the 1970s E. Mielke investigated a Klein-Gordon field ϕ with bicubic (order 6) potential

$$V(\phi) = m^2|\phi|^2(1 - \beta|\phi|^4) \, , \qquad \beta|\phi| \leq 1 \, . \tag{42}$$

He found that the corresponding nonlinear Klein-Gordon equation (with higher order self-interaction) allows for non-topological soliton solutions (Mielke 1978, Mielke 1979). Our three authors could draw upon

---

[55]Kim's linearly expanding phase of a Robertson-Walker model with JBD field sheds light on Scholz' approach, and vice versa. Embedded in a Weyl geometric context, Kim's model is nothing but a Minkowski-Weyl universe with flat spatial slices ($\kappa = 0$). If Kim's analysis of the pure scalar field dynamics without potential is correct, it tranfers to the Weyl geometric context after adaptation of the parameter.

[56]At the moment this is a minority position among physicists, best expressed by correspondents of the *Alternative Cosmology Group*, www.cosmology.info.

[57]Scale covariance of the scalar field does not play a role in this research; but it should be just a question of time until this further step is undertaken.

this result at least as a "toy model", as they admit, for their investigation of scalar fields as a potential source for *dark matter* (Fuchs e.a. 2006, 44). In an empirical investigation of their own Fuchs and Mielke show that the observed rotation curves of 54 galaxies stand in good agreement with expectations derived even from their toy model (Fuchs/Mielke 2004).

The work of the three authors demonstrates that the scalar field approach to "dark matter" is worthwhile to pursue. It is concentrated and goes deeply into empirical evidence. But the gap between scalar field halos and gravity remains still wide open. Some authors have started recently to tackle such questions based on JBD or Weyl-Dirac theory.

Hongsu Kim, whom we met already in the passage on dark energy, does so in (Kim 2007) by the Brans-Dicke approach. He uses a method developed already in the 1970s to construct axis symmetric solutions of the scalar field equation and the JBD version of Einstein equations with a singularity along the symmetry axis. He adds a highly interesting discussion of the singularity of the modified metric along the axis ($\theta = 0$). He remarks that this is a true singular direction, not only a coordinate singularity, and estimates the velocity of timelike trajectories close to the axis. He finds them to be close to the speed of light and arrives at the conclusion that the " bizarre singularity at $\theta = 0, \pi$ of the Schwarzschild-de Sitter-type solution in BD gravity theory can account for the relativistic bipolar outflows (twin jets) extending from the central region of active galactic nuclei (AGNs)" (Kim 2007, 24).

If this observation is right, even though only in principle, it will by far outweigh Kim's rough estimation of rotation velocities. The acceleration of jet matter is an unsolved riddle of astrophysics. It would be a great success for the approach, if a scalar field extension of Einstein gravity would be able to give a clue to this challenging phenomenon. Kim's analysis is formulated in classical JBD theory. He does not even consider conformal transformations to the Einstein frame, but rather stays in the Jordan frame. It will be interesting to see, what change it will make to take up conformal rescaling and Weyl geometric methods for this question.

None of the scalar field or Weyl geometric attempts to explain "dark matter" can yet compete in precision with Mannheim's conformal gravity approach. But in the range of proposals (in particular of Fuchs/Mielke/Schunck, Kim, and Castro) we have the seeds of an Ansatz for a promising research program. Whether it will lead to a solution of the problem remains, of course, to be seen.

5. Discussion

We have seen that JBD scalar tensor theory works in a Weyl geometric structure, although in most cases unknowingly.[58] The analysis of Ehlers/Pirani/Schild shows that Weyl geometry is deeply rooted in the basic structures of gravity. A first step towards founding gravity upon quantum physical structures (flow lines of WKP approximation of Dirac or Klein-Gordon fields) rather than on classical particle paths has been made by Audretsch/Gähler/Straumann.[59] Deeper links (Feynman path integral methods) or broader ones (geometrization of quantum potential) to quantum physics have not been discussed in this article.[60] But already the twin segments of theoretical physics considered here, elementary particle physics and cosmology, show remarkable features of Weyl geometry in recent and present physics.

There seem to be intriguing perspectives for Weyl geometric scalar fields, at least on a theoretical level, for approaching the problem of "mass generation" (as it is usually called) in particle physics. Englert, Smolin, Cheng, Rączka/Pawłowski and Drechsler/Tann have opened a view, not widely perceived among present physicists, of how a basic scalar field may participate in the mass generation of fermions and the electroweak bosons by coupling them to gravity. Their work has been marginalized during the rise of the standard model SMEP. But time may be ripe for reconsidering this nearly forgotten Ansatz.

From the Weyl geometric perspective, Drechsler's and Tann's analysis indicates most markedly a possible link between gravity and particle physics at an unexpectedly "low" energy level. This energy level will be reached by the LHC experiments startin November 2009. Already during the next five to ten years we shall learn more about whether the famous Higgs particle does indeed show up, at the end of the day, or whether the scepticism of scale covariant scalar field theoricians with regard to a massive Higgs particle is empirically supported in the long run.[61]

We also have found interesting aspects of the analysis of the "dark" matter problem by scale covariant scalar fields in the works of Kim, Castro, Fuchs/Mielke/Schunck and others. Scalar field models of spherically or axially symmetric solutions of the slightly generalized Einstein equation (and the Klein-Gordon equation) show properties which open promising perspectives for further investigations. The authors of these

[58]The scale connection φ in "Einstein frame" (scalar field gauge) is here usually hidden in partial derivative expressions equivalent to $\varphi = -d \log \phi = -d\omega$, where $\phi(x) = e^{\omega(x)}$ is the scalar field in "Jordan frame" (Riemann gauge).

[59](Ehlers/Pirani/Schild 1972), (Audretsch/Gähler/Straumann 1984).

[60]For the first question see (Narlikar/Padmanabhan 1983), for the second (Santamato 1985, Santamato 1984) and the succeeding literature. A recent survey on the last point is given in (Carroll 2005, Carroll 2004).

[61]Cf. (Kniehl/Sirlin 2000).

researches come from different directions and start to dig a land which seems worth the trouble of farming it more deeply. From our point of view, it it should be investigated whether introducing gauge transformations and Weylian scale connections into Kim's approach, or considering not only "trivial" Weylianizations (as we have called it above) in Castro's approach, helps to advance the understanding of the dark matter problem.

Most remarkable seem the structural possibilities opened up by Weyl geometry for analyzing how the scalar field energy tensor introduces a repulsive element into (scalar field extended) gravity, usually considered as vacuum or "dark" energy. The scalar field energy tensor allows for a much wider range of dynamical possibilities than usually seen in the framework of classical Friedmann-Lemaitre models. Even a balanced (non-expanding) spacetime geometry appears to be dynamically possible and, under certain assumptions, even natural. It is interesting to see that Kim's linearly expanding Robertson-Walker model, Masreliez' scale expanding cosmos, and Scholz' Weyl universes characterize one and the same class of spacetimes in the framework of Weyl geometry.

In addition, Weyl geometric gravity theory, with modified scale invariant Hilbert-Einstein action coupled to the scalar field, sheds new light on cosmological redshift. In this frame, the famous expanding space explanation of the Hubble redshift appears only as one possible perspective among others. From a theoretical point of view, it even need not be considered as the most convincing one.[62] With such questions we enter a terrain which physicists usually consider as morass; but Weyl geometry gives these investigations a safe conceptual framework.

Also here, like in the case of the Higgs mechanism, we have increasing observational evidence. It will contribute either to dissolving the standard wisdom or harden it against theoretically motivated scepticism. In the case of cosmological redshift, we look forward with great interest to more data on metallicity in galaxies and quasars. At present the indicators of a systematic development of metallicity in galaxies is doubtful, in quasars at best non-existent and apparently already refuted by empirical data.[63] An interesting debate on the reliability of the interpretation of the CMB anisotropy structure as indicator of primordial densitiy fluctuations has started.[64]

It will be interesting to see, whether in the years to come developments of scalar field explanations in cosmology harden and join in with new developments in high energy physics and observational cosmology;

[62]Remember Weyl's expectation that some day a "more physical" explanation will probably take the place of the "space kinematical" description of cosmological redshift. He considered the latter as of heuristic value only, due to its mathematical simplicity (Weyl 1930).

[63](Hasinger e.a. 2002), for a recent critical analysis see (Yang e.a. 2009).

[64]Cf. F. Steiner's contribution, this volume, and, among others, (Ayaita 2009).

or whether they fall apart without contributing markedly to a better understanding of the challenging phenomena of elementary particle physics and cosmology. At present there are good reasons to hope for the first outcome.

References

Adler, Stephen. 1982. "Einstein gravity as a symmetry-breaking effect in quantum field theory." *Reviews of Modern Physics* 54:729–766.

Audretsch, Jürgen; Gähler, Franz; Straumann Norbert. 1984. "Wave fields in Weyl spaces and conditions for the existence of a preferred pseudo-riemannian structure." *Communications in Mathematical Physics* 95:41–51.

Ayaita, Youness; Weber, Mark; Wetterich Christof. 2009. "Too few spots in the Cosmic Microwave Background." arXiv:090.3324.

Bergmann, Peter G. 1942. *Introduction to the Theory of Relativity.* Englewood Cliffs: Prentice Hall. Reprint New York: Dover 1976.

Bertotti, B.; Balbinot, R; Bergia, S.; Messina, A. 1990. *Modern Cosmology in Retrospect.* Cambridge: University Press.

Calderbank, D; Pedersen, H. 2000. Einstein-Weyl geometry. In *Surveys in Differential Geometry. Essays on Einstein Manifolds*, ed. C. Le Brun; M. Wang. International Press pp. 387–423.

Callan, Curtis; Coleman, Sidney; Jackiw, Roman. 1970. "A new improved energy-momentum tensor." *Annals of Physics* 59:42–73.

Carroll, Robert. 2004. "Gravity and the quantum potential." arXiv:gr-qc/0406004.

Carroll, Robert. 2005. "Fluctuations, gravity, and the quantum potential." gr-qc/0501045.

Cartier, Pierre. 2001. "La géométrie infiniésimale pure et le boson de Higgs." Talk given at the conference *Géométrie au vingtième siècle, 1930 – 2000,* http://semioweb.msh-paris.fr/mathopales/geoconf2000/videos.asp.

Castro, Carlos. 1992. "On Weyl geometry, random processes, and geometric quantum mechanics." *Foundations of Physics* 22:569–615.

Castro, Carlos. 2007*b*. "On dark energy, Weyl's geometry, different derivations of the vacuum energy density and the Pioneer anomaly." *Foundations of Physics* 37:366–409.

Castro, Carlos. 2009. "The cosmological constant and Pioneer anomaly from Weyl geometry and Mach's principle." *Physics Letters B* 675:226–230.

Cheng, Hung. 1988. "Possible existence of Weyl's vector meson." *Physical Review Letters* 61:2182–2184.

Coleman, Sidney; Weinberg, Erick. 1973. "Radiative corrections as the origin of sponteneous symmetry breaking." *Physical Review D* 7:1888–1910.

Dirac, Paul A.M. 1973. "Long range forces and broken symmetries." *Proceedings Royal Society London A* 333:403–418.

Drechsler, Wolfgang. 1977. *Fibre Bundle Techniques in Gauge Theories. Lectures in Mathematical Physics at the University of Austin.* Berlin etc.: Springer.

Drechsler, Wolfgang. 1999. "Mass generation by Weyl symmetry breaking." *Foundations of Physics* 29:1327–1369.

Drechsler, Wolfgang; Tann, Hanno 1999. "Broken Weyl invariance and the origin of mass." *Foundations of Physics* 29(7):1023–1064. [gr-qc/98020vv v1].

Earman, John. 2001. "Lambda: the constant that refuses to die." *Archive for History of Exact Sciences* 55:189–220.

Ehlers, Jürgen; Pirani, Felix; Schild, Alfred. 1972. The geometry of free fall and light propagation. In *General Relativity, Papers in Honour of J.L. Synge*, ed. Lochlainn O'Raiferteagh. Oxford: Clarendon Press pp. 63–84.

Englert, François; Gunzig, Edgar; Truffin C.; Windey, P. 1975. "Conformal invariant relativity with dynamical symmetry breakdown." *Physics Letters* 57 B:73–76.

Faraoni, Valerio. 2004. *Cosmology in Scalar-Tensor Gravity.* Dordrecht etc.: Kluwer.

Faraoni, Valerio; Gunzig, Edgard; Nardone, Paquale. 1998. "Transformations in classical gravitational theories and in cosmology." *Fundamentals in Cosmic Physics* 20:121ff. [arXiv:gr-qc/9811047].

Flato, Moshé; Raçka, Ryszard. 1988. "A possible gravitational origin of the Higgs field in the standard model." *Physics Letters B* 208:110–114. Preprint, SISSA (Scuola Internzionale Superiore di Studi Avanzate), Trieste, 1987 107/87/EP.

Flato, Moshé; Simon, J. 1972. "Wightman formulation for the quantization of the gravitational field." *Physical Review D* 5:332–341.

Folland, George B. 1970. "Weyl manifolds." *Journal of Differential Geometry* 4:145–153.

Fuchs, Bernhard; Mielke, Eckehard; Schunck Franz. 2006. "Dark matter halos as Bose-Einstein condensates." In *Tenth Marcel Grossmann Meeting on Recent Developments in Theoretical nd Experimental Relativity, Gravitation and Relativistic Field Theories.* singapore: World Scientific pp. 39–58.

Fuchs, Burkhard; Mileke, Eckehard. 2004. "Scaling behaviour of a scalar field model of dark matter halos." *Monthly Notices Royal Astronomical Society* 350:707–709. arXiv:astro.ph/0401575.

Fujii, Yasunori; Maeda, Kei-Chi. 2003. *The Scalar-Tensor Theory of Gravitation.* Cambridge: UP.

Gauduchon, P. 1995. "La 1-forme de torsion d'une variété hermitienne compacte." *Journal für die reine und angewandte Mathematik* 469:1–50.

Goenner, Hubert. 2004. "On the history of unified field theories." *Living Reviews in Relativity* 2004-2. [http://relativity.livingreviews.org/Articles/lrr-2004-2].

Hasinger, Günther; Schartel, N.; Komossa, Stefanie. 2002. "Discovery of an ionized Fe K edge in the $z = 3.91$ broad absorption line quasar APM 08279+5255 with XMM-Newton." *Astrophysical Journal* 573:L77–L80.
arXiv:astro-ph/0207321.

Hawking, Stephen; Ellis, George. 1973. *The Large Scale Structure of Space-Time.* Cambridge: University Press.

Hayashi, Kenji; Kugo, Taichiro. 1977. "Elementary particles and Weyl's gauge field." *Progress of Theoretical Physics* 57:431–440. [Erratum vol. 58, 681].

Hayashi, Kenji; Kugo, Taichiro. 1979. "Remarks on Weyl's gauge field." *Progress of Theoretical Physics* 61:334–346.

Hehl, Friedrich; Puntigam, Roland; Tsantilis, Efstatios. 1996. A quadratic curvature Lagrangian of Pawloski and Raczka: A finger exercise with MathTensor. In *Relativity and Scientific Computing ...*, ed. F.W. Hehl; R. Puntigam; H. Ruder. Berlin etc.: Springer. [gr-qc/9601002].

Hehl, Friedrich W.; McCrea, J. Dermott; Mielke, Eckehard; Ne'eman, Yuval. 1995. "Metric-affine gauge theory of gravity: Field equations, Noether identities, world spinors, and breaking of dilation invariance." *Physics Reports* 258:1–171.

Higa, T. 1993. "Weyl manifolds and Einstein-Weyl manifolds." *Commentarii Mathematici Sancti Pauli* 42:143–160.

Higgs, Peter. 1964. "Broken symmetries and the masses of gauge bosons." *Physical Review Letters* 13:508–509.

Israelit, Mark. 2002*a*. "Primary matter creation in a Weyl-Dirac cosmological model." *Foundation of Physics* 32:295–321.

Israelit, Mark. 2002*b*. "Quintessence and dark matter created by Weyl-Dirac geometry." *Foundation of Physics* 32:945–961.

Israelit, Mark; Rosen, Nathan. 1992. "Weyl-Dirac geometry and dark matter." *Foundations of Physics* 22:555–568.

Israelit, Mark; Rosen, Nathan. 1993. "Weylian dark matter and cosmology." *Foundations of Physics* 24:901–915.

Israelit, Mark; Rosen, Nathan. 1995. "Cosmic dark matter and Dirac gauge function." *Foundations of Physics* 25:763–777.

Israelit, Mark; Rosen, Nathan. 1996. "A Weyl-Dirac geometric particle." *Foundations of Physics* 26:585–594.

Kaiser, David. 2007. "When fields collide." *Scientific American* June, pp. 62–69.

Kao, W.F. 1990. "Inflationary solution in Weyl invariant theory." *Physics Letters A* 149:76–78.

Kim, Hongsu. 2005. "Brans-Dicke theory as a unified model for dark matter-dark energy." *Monthly Notices Royal Astronomical Society* 364:813–822. [astro-ph/0408577].

Kim, Hongsu. 2007. "Can the Brans-Dicke gravity with Λ possibly be a theory of dark matter?" astro-ph/0604055v3.

Kniehl, B.A.; Sirlin, A. 2000. "Mass scale of new physics in the absence of the Higgs boson." *European Journal of Physics C* 16:635–639.

Kragh, Helge. 2007. *Conceptions of Cosmos. From Myths to the Accelerating Universe: A History of Cosmology.* Oxford: University Press.

Livio, Mario. 2003. *The Dark Universe: Matter, Energy and Gravity. Proceedings of the Space Telescope Science Institute Symposium, Baltimor, April 2–5, 2001.* Cambridge: University Press.

Mannheim, Philip. 1994. "Open questions in classical gravity." *Foundations of Physics* 224:487–511.

Mannheim, Philip. 2000. "Attractive and repulsive gravity." *Foundations of Physics* 22:709–746.

Mannheim, Philip; Kazanas, Demosthenes. 1989. "Exact vacuum solution to conformal Weyl gravity and galactic rotation curves." *Astrophysical Journal* 342:635–638.

Masreliez, John. 2004*a*. "Scale expanding cosmos theory I — an introduction." *Apeiron* 11:99–133.

Masreliez, John. 2004*b*. "Scale expanding cosmos theory II — cosmic drag." *Apeiron* 11:1–29.

Mielke, Eckehard. 1978. "Note on localized solutions of a nonlinear Klein-Gordon equation related to Riemannian geometry." *Physical Reviews D* 18:4525–4528.

Mielke, Eckehard. 1979. "Mass formulas for solitons with quantized charge." *Lett. Nuovo Cimento* 25:423–428.

Mirabotalebi, S.; Jalaluzadeh, S.; Sadegh Mohaved, M.; Sepangi, H.R. 2008. "Weyl-Dirac theory predictions on galactic scales." gr-qc/0504117.

Moyassari, P.; Jalalzadeh, S. 2004. "Weyl geometry approach to describe planetary systems." gr-qc/0410073.

Narlikar, Jayant; Padmanabhan, T. 1983. "Quantum cosmology via path integrals." *Physics Reports* 100:151–200.

O'Raifeartaigh, Lochlainn. 1997. *The Dawning of Gauge Theory.* Princeton: University Press.

O'Raifeartaigh, Lochlainn; Straumann, Norbert. 2000. "Gauge theory: Historical origins and some modern developments." *Reviews of Modern Physics* 72:1–23.

Ornea, Liviu. 2001. "Weyl structures on quaternionic manifolds. A state of the art." arXiv:math/0105041.

Pawłowski, Marek. 1990. "Can gravity do what Higgs does?" Preprint IC/90/454.

Pawłowski, Marek; Rączka, Ryszard. 1994*a*. "Mass generation in the standard model without dynamical Higgs field." hep-th/9403303.

Pawłowski, Marek; Rączka, Ryszard. 1994*b*. "A unified conformal model for fundamental interactions without dynamical Higgs field." *Foundations of Physics* 24:1305–1327. ILAS 4/94 hep-th/9407137.

Pawłowski, Marek; Rączka, Ryszard. 1995*a*. "A Higgs-free model for fundamental interactions and its implications." Preprint. ILAS/EP-1-1995.

Pawłowski, Marek; Rączka, Ryszard. 1995*b*. "A Higgs-free model for fundamental interactions. Part I: Formulation of the model." ILAS/EP-4-1995.

Pawłowski, Marek; Rączka, Ryszard. 1995*c*. A Higgs-free model for fundamental interactions. Part I: Formulation of the model. In *Modern Group Theoretical Methods in Physics*, ed. J. Bertrand e.a. pp. 221–232. Preprint ILAS/EP-3-1995, hep-ph/9503269.

Pawłowski, Marek; Rączka, Ryszard. 1995*d*. "A Higgs-free model for fundamental interactions. Part II: Predictions for electroweak observables." ILAS/EP-4-1995.

Penrose, Roger. 1965. "Zero rest-mass fields including gravitation: asymptotic behaviour." *Proceedings Royal Society London A* 284:159–203.

Quiros, Israel. 2000*a*. "Transformations of units and world's geometry." gr-qc/0004014.

Quiros, Israel. 2000*b*. "The Weyl anomaly and the nature of the background geometry." gr-qc/00011056.

Rubin, V.C. 2003. "A brief history of dark matter." In (Livio 2003, 1–13).

Santamato, E. 1984. "Geometric derivation of the Schrödinger equation from classical mechanics in curved Weyl spaces." *Physical Review D* 29:216–222.

Santamato, E. 1985. "Gauge-invariant statistical mechanics and average action principle for the Klein-Gordon particle in geometric quantum mechanics." *Physical Review D* 32:2615 – 2621.

Scholz, Erhard. 2004. "Hermann Weyl's analysis of the "problem of space" and the origin of gauge structures." *Science in Context* 17:165–197.

Scholz, Erhard. 2005*a*. Local spinor structures in V. Fock's and H. Weyl's work on the Dirac equation (1929). In *Géométrie au XXième siècle, 1930 – 2000. Histoire et Horzions*, ed. D. Flament; J. Kouneiher; P. Nabonnand; J.-J. Szczeciniarz. Paris: Hermann pp. 284–301. [http://arxiv.org/physics/0409158].

Scholz, Erhard. 2005*b*. "On the geometry of cosmological model building." arXiv:gr-qc/0511113.

Scholz, Erhard. 2009. "Cosmological spacetimes balanced by a Weyl geometric scale covariant scalar field." *Foundations of Physics* 39:45–72. arXiv:0805.2557v3.

Scholz, Erhard (ed.). 2001. *Hermann Weyl's* Raum - Zeit - Materie *and a General Introduction to His Scientific Work*. Basel etc.: Birkhäuser.

Schunck, Franz E. 1995. *Selbstgravitierende bosonische Materie*. Göttingen: Cuvillier Verlag.

Schunck, Franz E. 1999. Boson halo: Scalar field model for dark halos of galaxies. In *Proceedings Eights Marcel Grossmann Meeting on General Relativity, Jerusaem 1997*, ed. T. Piran; R. Ruffini. Singapore: Word Scientific pp. 1447–1449.

Shojai, Fatimah; Shojai, Ali. 2002. "Weyl geometry and quantum gravity." Max Planck Institute for Gravitational Physics *Preprint* AEI-2002-060 [gr-qc/0306099].

Sigurdsson, Skúli. 1991. "Hermann Weyl, Mathematics and Physics, 1900 – 1927." Cambridge, Mass.: PhD Dissertation, Harvard University.

Smolin, Lee. 1979. "Towards a theory of spacetime structure at very short distances." *Nuclear Physics B* 160:253–268.

Straumann, Norbert. 1987. "Zum Ursprung der Eichtheorien." *Physikalische Blätter* 43:414–421.

Tann, Hanno. 1998. *Einbettung der Quantentheorie eines Skalarfeldes in eine Weyl Geometrie — Weyl Symmetrie und ihre Brechung.* München: Utz.

Trimble, Virginia. 1990. "History of dark matter in the universe (1922-1974)." In (Bertotti 1990, 355–364).

Utiyama, Ryoyu. 1973. "On Weyl's gauge field." *Progress of Theoretical Physics* 50:2028–2090.

Utiyama, Ryoyu. 1975*a*. "On Weyl's gauge field." *General Relativity and Gravitation* 6:41–47.

Utiyama, Ryoyu. 1975*b*. "On Weyl's gauge field II." *Progress of Theoretical Physics* 53:565–574.

Vizgin, Vladimir. 1994. *Unified Field Theories in the First Third of the 20th Century.* Translated from the Russian by J. B. Barbour. Basel etc.: Birkhäuser.

Wei, Hao; Cai, Rong-gen. 2007. "Cheng-Weyl vector field and its cosmological application." *Journal of Cosmology and Astroparticle Physics* (15). [astro-ph/0607064].

Weinberg, Stephen. 1967. "A model of leptons." *Physical Review Letters* 19:1264–1266.

Weyl, Hermann. 1918*a*. "Gravitation und Elektrizität." *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* pp. 465–480. GA II, 29–42, [31]. In (Weyl 1968, II, 29–42) [31], English in (O'Raifeartaigh 1997, 24–37).

Weyl, Hermann. 1918*b*. *Raum, - Zeit - Materie.* Berlin etc.: Springer. Weitere Auflagen: ^{2}1919, ^{3}1919, ^{4}1921, ^{5}1923, ^{6}1970, ^{7}1988, ^{8}1993. English and French translations from the 4th ed. in 1922.

Weyl, Hermann. 1918*c*. "Reine Infinitesimalgeometrie." *Mathematische Zeitschrift* 2:384–411. In (Weyl 1968, II, 1–28).

Weyl, Hermann. 1922. *Space – Time – Matter.* Translated from the 4th German edition by H. Brose. London: Methuen. Reprint New York: Dover 1952.

Weyl, Hermann. 1930. "Redshift and relativistic cosmology." *London, Edinburgh and Dublin Philosophical Magazine* 9:936–943. In (Weyl 1968, III, 300–307).

Weyl, Hermann. 1968. *Gesammelte Abhandlungen, 4 vols.* Ed. K. Chandrasekharan. Berlin etc.: Springer.

Wheeler, James. 1990. "Quantum measurement and geometry." *Physical Review D* 41:431–441.

Wu, Cue-Liang. 2004. "Conformal scaling gauge symmetry and inflationary universe." *International Jorunal of Modern Physics A* 20:811ff. astro-ph/0607064.

Yang, Rong-Jia; Zhang, Shuang Nan. 2009. "Age crisis in ΛCDM model?" arXiv:0905.2683.

Zee, Anthony. 1979. "Broken-symmetric theory of gravity." *Physical Review Letters* 42:417–421.

Zee, Anthony. 1982*b*. "A theory of gravity based on the Weyl-Eddington action." *Physics Letters B* 109:183–186.

Zee, Anthony. 1983. "Einstein gravity emerging from quantum Weyl gravity." *Annals of Physics* 151:431–443.