Hermann Weyl and Large Numbers in Relativistic Cosmology

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1. Introduction

Since time immemorial philosophers, poets, and scientists have pondered the relationship between the micro- and the mega-world. The relevant scale, what counts as the micro- and the mega-, has always been determined by the scientific knowledge of the time. Since Newton, the scales of the largest and the smallest have extended by ten orders of magnitude in both directions. Equally strikingly, the meanings of 'micro' and 'mega' have changed in the historical development from the unification of celestial and terrestrial mechanics, to the physical study of stars by means of spectral analysis, to the micro-physical explanation of the baryon asymmetry of the universe.

It was only in the late 1910s, however, that the first physical fact was discovered that could provide a quantitative clue to the interconnection between the micro- and mega-worlds. It was a famous mathematician, Hermann Weyl, who made this discovery.

His discovery later gave rise to such different ideas as the hypothetical variation of the gravitational constant and the anthropic principle. More cautiously, it was referred to as "an unexplained empirical connection between meta-galactic parameters and micro-physical constants" (Zel'manov 1962, p. 496).

Although this link between the micro- and mega-worlds is regarded as an empirical fact, its recognition was intertwined with developments in advanced theoretical physics. Before turning to the circumstances of the discovery of this fact, let us look at its contemporary status, which clearly points to its empirical nature.

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2. The Cosmological and Micro-physical Parameters

The universe as a whole is the largest physical object that can be described by a limited number of parameters. The basic mega-parameters are the average density of mass (or energy), $\rho \approx 10^{-31}$ g/cm^3, and the Hubble constant, $H \approx 2\times10^{-18}$ s^{-1}, which describes the rate of the expansion of the universe by relating the speed of the recession of galaxies to the intergalactic distance according to $v = Hr$. The gravitational constant G and the velocity of light c can also be attributed to mega-physics.

The characteristic parameters of micro-physics are the mass of an elementary particle m, the elementary charge e, the Planck constant $\hbar$, and the velocity of light c. The uncertainty of a few orders of magnitude in "the" mass of a particle does not really matter in the present context. For definiteness, we shall assume it to be the electron mass. Thus, mega-physics is represented by the quantities H, ρ, G, and c, while micro-physics is represented by m, e, $\hbar$, and c. Let us also put aside the question of whether one should use the coupling constant of the strong or weak interaction in place of e: in the 1920s, no such question existed.

Only dimensionless quantities, the values of which are independent of the units of measurement, can be regarded as being of genuine theoretical interest. Among such quantities, constructed from the parameters

$$H, \rho, G, m, e, \hbar, c \tag{1}$$

are three combinations whose values lie very close to the unusually large number 10^{40}. One of them is the ratio of the strengths of electromagnetic and gravitational interactions

$$Q_1 = e^2/Gm^2 = 4\times10^{42} \approx 10^{40}. \tag{2a}$$

Two other combinations can be obtained by taking the ratio between the distance and density scales characteristic of the micro- and macro-worlds: between the so-called radius of the universe, $R = c/H$, and the classical electron radius, $r_e = e^2/mc^2$, and also between ρ and m/r_e^3:

$$Q_2 = R/r_e = 3\times10^{40} \approx 10^{40}, \tag{2b}$$

$$Q_3 = mr_e^3/\rho = 3\times10^{40} \approx 10^{40}. \tag{2c}$$

The fact that these independent dimensionless quantities, Q_1, Q_2, and Q_3, are all so close to the large number 10^{40},

$$Q_1 \approx Q_2 \approx Q_3 \approx 10^{40}, \qquad (3)$$

is called the *coincidence of large numbers*. This coincidence turns out to be a "purely" empirical fact that is not incorporated in any theory. (The quotation marks here are intended to remind the reader that it is only our enlightened age that permits us to regard such quantities as the universe's average density and the rate of its expansion as being empirical.)

Since the Hubble constant, H, appears in relations (2) and (3), one might think that the coincidence of large numbers came to be known only after 1929, when Hubble's law was discovered. This is not true: The coincidence came to the attention of physicists in the early 1920s and Arthur Eddington was one of its most active proponents. To clarify these circumstances, let us turn to the history of relativistic cosmology, with which the history of "large numbers" is closely connected.

3. The First Steps of Relativistic Cosmology

Relativistic cosmology was born in Einstein's 1917 paper, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," in which he suggested the first cosmological model based on general relativity (GR). Einstein's universe was described by two parameters, the radius of curvature R and the density of matter ρ. Einstein did not regard his model as a purely theoretical construct. For him, its most essential features, such as homogeneity, isotropy, constancy in time, and non-zero average density, were experimental and observational facts. Evidently, he knew nothing about the galactic structure of the universe (it was no more than a hypothesis then), nor about Vesto Slipher's discovery in 1913 of the enormous velocities of some galaxies (referred to, at that time, as "spiral nebulae"), which later developed into Hubble's law. Einstein supported his assumption about the static nature of the universe by referring to "the most important fact that we draw from experience as to the distribution of matter . . . that the relative velocities of the stars are very small as compared with the velocity of light" (1917, p. 148, trans. by W. Perrett and G. B. Jeffery). Einstein never specified which "experience" he had in mind. One thing is clear: this fact was of signal importance to Einstein. He mentioned it seven times in the eleven-page paper.

Einstein's acceptance of the static nature and the non-zero density of the universe forced him to generalize the field equations of GR by introducing the cosmological constant Λ:

$$R_{ik} - \tfrac{1}{2} g_{ik} R = \kappa T_{ik} - \Lambda g_{ik}, \tag{4}$$

which led him to assume a closed spatial geometry and the relation

$$1/R^2 = \kappa\rho/2 = \Lambda, \tag{5}$$

connecting the quantities R and ρ.

Einstein himself never attempted to estimate R and ρ on the basis of astronomical data: “Whether, from the standpoint of present astronomical knowledge, it [i.e., Einstein’s cosmological model] is tenable, will not be here discussed” (1917, p. 152; trans. by W. Perrett and G. B. Jeffery).

Willem de Sitter was the first to do this, in the same year, 1917. In his third (and last) paper in a series that discussed Einstein’s gravitation theory and its astronomical consequences, he turned to the cosmological problem (de Sitter 1917). Having described Einstein’s cosmological model, he suggested another model known today as the de Sitter model, in which the matter density is zero while space, depending on the sign of Λ, can be either open or closed and the curvature radius is related to Λ:

$$3/R^2 = \Lambda. \tag{6}$$

De Sitter concluded his paper with an estimate of R based on the astronomical data.

The Einstein model allowed two types of test: by deriving geometrical consequences for the curvature (for example, for the behavior of light rays) and by obtaining the density from equation (5). De Sitter’s model, on the other hand, permitted only geometrical considerations to be used, as it assumed $\rho = 0$.

De Sitter relied on much more extensive astronomical data than the single fact that the stars are at rest. He took into account the galactic structure of the universe, the size and mass of our Galaxy, and even the recently discovered shifts in the spectra of three galaxies (“spiral nebulae”). He used Schwarzschild’s (1900) work, which compared astronomical data with assumptions about spatial curvature. Based, of course, on pre-relativistic considerations, Schwarzschild put a constraint on the curvature radius from below. For example, he noted that the light from the other side of the sun must be absorbed as it travels “around the world.” (In a transparent, spatially-closed universe, any object can be observed in two opposite directions).

A constraint on the curvature radius from above was more interesting. In Einstein's cosmological model, this constraint is based on the estimate of the average matter density in the universe. De Sitter used the available data on the density of stars at the center of our galaxy and on its size (about 10^4 parsecs), and he took inter-galactic distances to be approximately equal to our galaxy's size. The result was

$$\rho > 10^{-27} \text{ g/cm}^3, \tag{7}$$

and, according to (5), the curvature radius is constrained by

$$R < 10^{27} \text{ cm}. \tag{8}$$

Assuming that space is closed and that the sun's reverse side is invisible, the estimate of R from below gave $R > 4 \times 10^{24}$ cm, while $R > 10^{25}$ cm follows from sufficiently small angular dimensions of the "spiral nebulae" and the hypothesis that their linear size is of the same order as that of our galaxy. Although de Sitter emphasized that these estimates were very crude, he believed that their agreement is remarkable and could not be expected a priori.

This is how the first quantitative cosmological parameters based on real observational data appeared. They claimed to describe the universe as a whole giving, for its radius

$$R \approx 10^{27} \text{ cm}, \tag{9}$$

and for the average density

$$\rho \approx 10^{-27} \text{ g/cm}^3. \tag{10}$$

Notably, despite all the difference between Einstein's model and present-day cosmology (today, the concept of the "radius of the universe" does not presuppose closed space), contemporary estimates of R and ρ differ from (9) and (10) only by a few orders of magnitude.

Thus, as early as in 1917, the coincidence of large numbers (2b) and (2c) could have been noticed. But no theoretical framework was available to account for it.

4. An Empirical Fact Discovered by a Mathematician

Hermann Weyl made a step towards discovering a “coincidence” of large numbers while working on his unified field theory (Weyl 1918). This takes us from observational astronomy to theoretical physics, or even to “purely” theoretical physics, as Weyl’s theory was never developed beyond the state of a “draft theory,” a mathematical construct that could not be tested by experiment. (This did not render it fruitless, for the notion of gauge symmetry, a key ingredient of contemporary physics, was first suggested by Weyl’s theory; see Vizgin 1994.)

The notion of spatial-temporal standards, or scales, served as the starting point in Weyl’s theory. In Einstein’s general relativity, which Weyl sought to generalize, the space-time metric

$$ds^2 = g_{ik} dx^i dx^k$$

is brought into correspondence with experiment by means of definite standards of length associated with solid bodies or light signals. This makes it possible to compare the lengths of metric intervals measured at different points of space-time.

Weyl suggested that, in accordance with the principle of locality, the lengths should be strictly comparable only at individual space-time points. This led him to a geometry that, while generalizing Riemannian geometry, described the properties of space-time, not only by ten components of the metric tensor $g_{ik}(x)$, but also by four quantities $\varphi_i(x)$ that could play the role of the vector potential of the electromagnetic field.

Besides arbitrary coordinates, Weyl’s theory also involved the so-called gauge, or scale, transformation

$$g'_{ik} = \lambda g_{ik}, \quad \varphi'_i = \varphi_i + \partial_i \lambda,$$

where $\lambda(x)$ is an arbitrary function of the space-time coordinates. This transformation was interpreted as a change of scale or the measuring standard (which, according to this theory, should be chosen at each point of space-time), since $ds'^2 = \lambda ds^2$.

Invariance under scale transformations was thus the pivoting point of Weyl’s theory. This led Weyl to field equations that cannot be transformed into Einstein’s equations in the limit $\varphi_i(x) = 0$ (the Lagrangian $R\sqrt{g}$, which leads to Einstein’s equations, is not scale invariant; therefore Weyl adopted, as his Lagrangian, the square of the curvature, $R_{j\,kl}^{\,i} R_{i}^{\,j\,kl} \sqrt{g}$). To defend this feature of the theory, Weyl noted in 1918:

> [I]t is highly improbable that Einstein's equations for the gravitational field are strictly valid. First and foremost, it is improbable because the gravitational constant is out of place among other natural constants, so that the gravitational radii of the charge and the mass of the electron have values of quite different orders of magnitude than, for example, the radius of the electron itself (they are smaller than the latter—the first by 10^{20} and the second by 10^{40}). (Weyl 1918, p. 476)

Here Weyl repeats his own remark from an article written in 1917 entirely from the point of view of GR. Having found a solution for an electrically charged point mass (also known as the Reisner–Nordström solution), Weyl introduced the lengths $r_{gm} = Gm/c^2$ (gravitational radius of mass m) and $r_{ge} = e\sqrt{G}/c^2$ (gravitational radius of the electric charge e). Comparing these lengths with the "electron radius," $r_e = e^2/mc^2$, he noted that, for the electron, $r_{ge}/r_e = 10^{20}$ and that this solution could hardly be used to understand the physics of the atom, since the gravitational field's influence would be important, in the case of the electron, only at a distance of the order $r_{ge} \approx 10^{-33}$ cm (Weyl 1917, p. 145). In 1918, Weyl also mentioned that "in the most general case, for a world with [the curvature scalar] $R \neq 0$, one can obtain, by appropriately choosing an arbitrary unit, $R = \text{const} = \pm 1$" (Weyl 1918, p. 475).

It seems that further elaboration of these findings led Weyl, for the first time, to turn to the coincidence of large numbers in his article of 1919.

The lack of a physical interpretation of the procedure of "choosing an arbitrary unit of length" was, from the physical point of view, the Achilles' heel of Weyl's theory. (It was precisely this point that prompted Einstein's critical comment, which, however, failed to convince Weyl.) Physics had always attached importance to the notion of measurement standards, and the standards and scales employed had invariably been presupposed to be as stable as possible. Physics had not recognized arbitrary spatio-temporal changes of scale. True, there was a certain arbitrariness in choosing the standards, but it had a discrete, or global, character. Having introduced an arbitrariness in calibrating the scales and being convinced that this represented a true realization of the principle of locality (as distinct from Einstein's approach, which was "half-hearted and inconsistent"), Weyl had to eliminate this arbitrariness one way or the other, so that his theory could be experimentally tested.

Weyl failed to find a convincing solution to this problem, but in his search for it he stumbled upon the values of length that could be regarded as fundamental characteristic scales. In 1918, he returned to comparing the quantities r_{ge}, r_{gm}, and r_e, his "official" goal being to demonstrate that Einstein's theory of gravitation could not serve as the basis of atomic physics.

In his 1919 paper, Weyl suggested a new variant of a unified theory that generalized GR and was based on a geometry that he introduced (i.e., the *Weyl geometry*). In this new theory, the gravitational Lagrangian remained the square of the curvature. The Maxwellian part of it, however, was no longer emerging in a natural way but was added "by hand." Through artful manipulations, Weyl managed to bring this Lagrangian into a quasi-Einsteinian form. This led him to a variational principle that included the Einsteinian component (with the cosmological term), the Maxwellian component, and a non-Maxwellian component, the square of the vector potential $\varphi_i\varphi^i$ ("the simplest expression found in the Mie theory" (Weyl 1919, p. 122), a theory that claimed to provide a unified field description of the electron and the electromagnetic field).

This "spoiled" both the Maxwellian and Einsteinian equations in the new version of the theory. However, according to Weyl, there was no contradiction with experiments (having to do, in the first place, with electromagnetism), since the non-Maxwellian term entered the equations with a very small factor of the order $1/R^2$, where R is the radius of the universe. (It seems that Weyl assumed a connection between the value of the cosmological constant and the radius of the universe, $\Lambda \approx 1/R^2$, figuring in Einstein's cosmological model.) By following this path, Weyl arrived at a unit of charge, the gravitational radius of which, $G^{1/2}e/c^2$, has the same order of magnitude as the radius of the universe. Noting that, in his work, the unit of electricity and the unit of action both have "cosmic values," Weyl emphasized: "*The 'cosmological' term that Einstein first added to his theory is a natural consequence of our original principles*" (Weyl 1919, p. 124, italic in the original).

In this way, Weyl clearly demonstrated, in 1919, his cosmological tendency, which had been absent in his previous works, and supplemented the microscopic quantities r_e, r_{ge}, and r_{gm} with a megascopic quantity, the radius of the universe R. It is in this paper, in discussing the "problem of matter," that Weyl, probably for the first time, pointed to

> the fact that, for the electron, there are dimensionless numbers the order of magnitude of which differs from unity by a great degree. Such is the relation between the radius of an electron and the gravitational radius of its mass,

which is equal to a magnitude of the 10^{40} order. The ratio of the electron radius to the radius of the universe may be of the same order. (Weyl 1919, p. 129)

What he has in mind here is the relation

$$\frac{r_e}{r_{gm}} \left(= \frac{e^2/mc^2}{Gm/c^2} = \frac{e^2}{Gm^2} \right) \approx \frac{R}{r_e}, \tag{11}$$

that is, the "coincidence of large numbers." Weyl's essay does not mention any specific numerical value for R, nor is there any reference to works that mention it. But Equation (11) suggests that Weyl adopted de Sitter's estimation $R \approx 10^{27}$ cm. In 1923, in the same context, Weyl put the radius of the universe at about 10^9 light years ($\approx 10^{27}$cm) (Weyl 1923, p. 323).

It is hard to tell if it was actually Weyl who, in that article (Weyl 1919), discovered, for the *first time*, the "coincidence of large numbers." The apropos tone of his remark suggests that this fact could already have been known to Weyl and was referred to as something curious. In any case, this fact could not originate before de Sitter made his astronomical estimates of R in 1917.

Weyl's 1919 paper may produce an impression that relation (11) *follows* from his theory. However, neither at that time, nor later, did his theory reach the stage at which it could be compared with observational data. Relation (11) was just *consistent* with Weyl's ideas. Indeed, some of his claims seem dubious; for example, the specific value for the radius of the universe R adopted in (11) was obtained by de Sitter in the framework of Einstein's cosmology and it was unclear whether this result could be imported into Weyl's theory.

Weyl's transition from his original Lagrangian, which was quadratic in the curvature and produced a field equation of the *fourth* order, to the *second*-order equations and his observations regarding a possible relationship between the electron and the universe are inferences that appear even more precarious. From a mathematical point of view, Weyl's manipulations were never properly justified (see Bergmann 1942, Pauli 1958). The most important unresolved problem was to effect the correspondence between the new theory and Newtonian gravitation, and to explain how "half" of the initial conditions (the second and third derivatives at time zero) in the Newtonian limit could be ignored. This explains why Dirac, who turned to Weyl's geometry fifty years later, avoided this difficulty by introducing an additional scalar field; he used it to construct a scale-invariant Lagrangian that was linear in the curvature and that led, consequently, to equations of the second order (Dirac 1973).

Despite the circumstances mentioned above, Equation (11) reinforced by Weyl's ideas was perceived as suggesting a unified approach to both micro-physics and cosmology. A. S. Eddington, the most enthusiastic adherent of this approach, believed that the "coincidence of large numbers" did follow from Weyl's theory. In his *Space, Time and Gravitation*, the introduction to which is dated May 1920, he described the relationship between electricity and gravitation wholly in terms of Weyl's unified theory:

> [Weyl's] theory suggests a mode of attacking the problems of how the electric charge of an electron is held together; at least it gives an explanation of why the gravitational force is so extremely weak compared with the electric force. It will be remembered that associated with the mass of the sun is a certain length, called the gravitational radius, which is equal to 1.5 kilometers. In the same way the gravitational mass or radius of an electron is 7×10^{-56} cms. Its electrical properties are similarly associated with a length 2×10^{-13} cms., which is called the electrical radius. The latter is generally supposed to correspond to the electron's actual dimensions. The theory suggests that the ratio of the gravitational to the electrical radius, 3×10^{42}, ought to be of the same order as the ratio of the latter to the radius of curvature of the world. This would require the radius of space to be of the order 6×10^{29} cms., or 2×10^{11} parsecs, which though somewhat larger than the provisional estimates made by de Sitter, is within the realm of possibility. (Eddington 1920, pp. 178–179)

Later, when it became clear that Weyl's ideas failed to produce a viable theory, Eddington turned to other theoretical constructs. Still, his central motivation had always been the search for a unity of micro- and mega-physics. Equation (11) frequently figured in his books and articles and was probably the least speculative of his arguments.

The meaning of R in relationships such as (11) changed radically with the emergence of Friedmann's and Lemaître's evolutionary models and especially after Hubble's discovery of the redshift law in 1929. Redshifts were immediately interpreted as a result of the universe's expansion. Consequently, R (the radius of the universe) could no longer be firmly associated with a closed cosmological model, although "aesthetic" preferences for this model survived for a long time. The value of the Hubble constant H obtained from observations determines the characteristic cosmological distance $R = c/H$, although a precise geometrical meaning of this quantity can only be specified within a particular cosmological model.

The "coincidence of large numbers" retained its importance, since the Hubble radius, $R = c/H$, proved to be close to the Einsteinian radius of the universe estimated by de Sitter. But the meaning of R changed: it was now a function of time $R(t)$, not a constant of nature. This undermined the belief in an underlying connection between the mega- and microscopic parameters. As a result, the "coincidence of large numbers" became an empirical relation and ceased to depend on purely theoretical speculations. At the same time, it became even more of an enigma. Indeed, two large numbers coincide, one of which, according to the theory, is constant (e^2/Gm^2), while the other depends on time ($R(t)/r$)

$$Q_1 = Q_2(t). \qquad (12)$$

Two totally different approaches were suggested for interpreting the "coincidence of large numbers" understood in this way. The first, initiated by Dirac (1937), attempted to find a new physical theory in which the value of $Q_1 = e^2/Gm^2$ would be dependent on time $Q_1 = Q_1(t)$, so that the equality $Q_1(t) = Q_2(t)$ would always hold. Since Dirac associated the dependence of Q_1 on time with the gravitational constant, $G = G(t)$, one would expect a new gravitational theory to be closely connected with cosmology, as the gravitational constant became bound up with the cosmological parameters H and ρ. Although this project led to some further interesting physical ideas, by itself, it failed to produce a viable physical theory.

The second interpretation of coincidence (12), which emerged two decades after the first one, does not presuppose the variation of physical constants with time. Relation (12) is regarded as an equation that determines a certain moment of time, or, more exactly, a certain epoch, namely the present cosmological epoch. The manner of this determination resurrects (in a new form) the anthropocentric approach to the universe and leads to challenging questions.

5. New Anthropocentrism in Cosmology

The anthropocentric approach to the "coincidence of large numbers" originated in two papers published by Robert Dicke in 1957 and 1961. Dicke's theoretical ideas were far from consistent. He expressed doubts that GR had a reliable experimental basis (Dicke 1957a). He believed that the equivalence principle was invalid for "weak interactions" (he used the term to refer both to the gravitational force and to Fermi's concept of weak interaction). Dicke suggested, as a consequence of his approach, that the constants of these interactions depended on time and space. He tried to formulate a new theory of gravitation (Dicke 1957b) that he regarded as a manifestation of electromagnetism with variable permeability of the vacuum.

Dicke thought that the "coincidence of large numbers" was sure evidence that the gravitational constant changed with time. He offered, in passing, his explanation for the large values of coinciding numbers. Being complex physical structures, human observers could not have evolved within the time characteristic of the atomic scale. The epoch of man should be described by a time that is both large and random with respect to that scale (see Dicke 1957a, p. 356). In discussing the variability of physical constants, Dicke insisted that "the age of the Universe 'today' is not fortuitous. It is biologically determined." He argued that, if the fine-structure constant had been much less or much larger than its contemporary value, the stars (the luminosity of which is highly sensitive to the value of this constant) would still have been, or would already have been, so cold that human existence would have been impossible (Dicke 1957b, p. 375).

In 1961, Dicke put forward a more detailed anthropocentric explanation of the "coincidence of large numbers": Since the elements heavier than hydrogen were indispensable to life ("to create physicists we need carbon") and were formed inside the stars, the epoch of humankind could be determined by the life span of main-sequence stars. This life span could approximately be expressed in terms of the physical constants

$$T \approx \left(\frac{m_p}{m}\right)^{5/2} \left(\frac{e^2}{\hbar c}\right)^3 \left(\frac{Gm_p^2}{\hbar c}\right)^{-1} \left(\frac{\hbar}{m_p c^2}\right),$$

which, with the accuracy of $(e^2/\hbar c)\,(m/m_p)^{1/2} \approx 10^{-4}$, represents the coincidence of large numbers (Dicke 1961).

Whereas the anthropic approach to the coincidence of large numbers is regarded today as an alternative to the hypothesis of the variability of physical constants, Dicke never separated these two ideas. Furthermore, at the very same time (i.e., around 1961), he was working on the scalar-tensor theory of gravitation in which the gravitational "constant" was a scalar field.

It is tempting to trace the currently popular anthropocentrism in physics and cosmology to Boltzmann's fluctuation cosmological hypothesis. Back in the 1930s, Matvei Bronstein and Lev Landau offered a distinctively anthropic description (in order to refute it), according to which large cosmological fluctuations were a necessary prerequisite of the existence of observers, that is, a *sine qua non* of human existence (see Gorelik and Frenkel 1994, §5.5).

Despite all the arguments that endow the physical properties of the universe with anthropocentric interpretations, the anthropic principle belongs today to metaphysics (in the original sense of the term), rather than to physics. The principle adds a genetic connection to the link between the observer and the object of observation. If the object is the universe itself, it should be possible for the observer to emerge within it; that is to say, "our universe is what it is because we were able to appear in it."

This should not be taken to mean that the anthropic principle has no place in physics. It provides an independent reason to postulate a super-universe consisting of numerous separate domains. Our universe, which proved itself capable of creating human observers, is one of them (see, e.g., Okun 1991, 1996).

6. Conclusion. A Mathematician in Physics

Mathematician Hermann Weyl found himself a place in the history of fundamental theoretical physics in the twentieth century. His interest in physical problems, his concrete results, fruitful ideas, and outstanding books on general relativity and quantum mechanics set him apart from the mathematicians of his generation. One might say that, in this respect, he inherited the role of Henri Poincaré.

The difference between mathematics and physics, despite their close and mutually beneficial interaction, is indeed great. What Weyl wrote about our knowledge of the physical world was a graphic illustration of a *mathematician's view* of such knowledge.

One of the earliest images of science's omnipotence—"Give me a firm point, and I will turn the world around"—invites one to view mathematics as the lever and to saddle theoretical physics with the task of looking for the Archimedean point. The search has to proceed in a realm that cannot, in principle, be mathematized, the realm of physical reality. It demands considerable physical intuition, possible only in those who have extensive experience of this reality. A mathematician who spent his life creating various levers will find physical reality—with its apparently chaotic collection of phenomena, facts, properties, and its mathematical eclecticism—to be rather alien. This impression persists even if the mathematician is able to find the notion of a lever that can be used to "turn the world around."

This alienation and resulting frustration can be easily perceived in a dialogue of two apostles of relativism recounted by Weyl (1924). The ideas of Mach discussed in the dialogue were instrumental in formulating the general theory of relativity. So far, however, no one has managed to transform Mach's principle from a verbal into a mathematical form. Cases in which an illuminating idea burns to the ashes are not infrequent in the history of physics: the light from burning often helps one to make some steps forward, which would otherwise have to be made in complete darkness. In the history of mathematics, more often than not, it is fortuitous and unnecessary parts of the original idea that are destroyed by the cleansing fire; a mathematical structure of beauty comes out of it to take its proper place.

Several years divide Weyl's two papers, (1924) and (1931). In fact, they belong to two different epochs in theoretical physics. In 1924, the hope of creating a unified geometrical field theory ran high. It was stimulated by Weyl's own work, which resulted in the first such theory suggested in 1918. By the early 1930s, the unsuccessful program of unified theories had withered under the impact of quantum theory. Nobody was more resolute in proclaiming the end of the epoch than Weyl himself when, in 1931, he called the project of unified theories "geometrical trinkets," allowing the community of physicists to choose a proper funeral rite. One can even suspect Weyl of nursing a grudge against physics for its rejection of the goal to which he had pointed, namely, the idea of locality, more consistently than it was done in Einstein's general relativity.

Cosmology was another significant element in Weyl's papers of 1924 and 1931. Both give one a good sense of the highly speculative nature of relativistic cosmology in the pre-Friedmann era. The earlier of the two papers should be dated to this stage as well, since nobody, including Einstein, paid any attention to Friedmann's achievements. It is much more amazing that, in 1931, Weyl still failed to notice a new stage in the development of relativistic cosmology, after Lemaître had published his works and Hubble had made his discovery, and when the expansion of the universe had already become an observational fact. As a mathematician, Weyl could afford to ignore developments in astrophysics. Yet, there was a more profound reason for his neglect.

It was Weyl the mathematician who tried, in 1919, to connect his unified theory with physical reality. He was the first to pay attention to the empirical fact of the "coincidence of large numbers," which he regarded as providing a connection between cosmology and micro-physics. The manner in which Weyl argued for this connection lacked mathematical (and even physical) rigor; he was driven by his desire to implement the unification project. He abandoned the project, yet the result he obtained survived and acquired a life of its own. The majestic idea about possible links between cosmology and micro-physics fascinated Eddington and drew Dirac far away from physics. No wonder that Weyl, the philosopher and mathematician who had given birth to this idea, was too much involved with it to pay attention to a non-static cosmology that was not part of his unified picture of the world.

Non-static cosmology is, in fact, absent from Weyl's later book (Weyl 1949). This can hardly be explained by the empirically unsatisfactory state of cosmology at that time (Hubble's estimate of the age of the universe was in conflict with geo- and astrophysical data). More likely than not, this absence can be explained by Weyl's general philosophical and mathematical approach to the physical representation of the world. The coincidence of large numbers, discredited by Eddington's ambitious yet barren constructs, received a totally different treatment in Weyl's works. He denied that gravitation was a fundamental property and tried to explain it as a certain kind of statistical effect of a large number of particles in the universe, in line with Mach's principle. This attempt remains nothing more than a fact of Weyl's biography.

There is no need to regret this, however. Under more or less similar circumstances, Boltzmann had the following to say (about his fluctuation cosmology):

> Certainly no one will think that this and similar [cosmological] speculations are important discoveries. No one will agree with the ancient philosophers that they are science's ultimate aim. Yet, does one have the right to ridicule them as totally devoid of any significance? Probably they are extending our horizons, making our thinking more flexible, and contributing to our knowledge of reality. (Boltzmann 1898, p. 90).

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