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Michel Janssen

The Einstein-Besso Manuscript: A Glimpse Behind the Curtain of the Wizard

Michel Janssen
Program in History of Science and Technology

In my segment of the course, I want to show you slides of some pages of a set of research notes produced by Albert Einstein (1879–1955) and his closest friend and confidant Michele Besso (1873–1955). These notes, known collectively as the Einstein-Besso manuscript, are from the period 1913–1914, when Einstein was still developing his general theory of relativity. Einstein first published the theory in the form in which physicists still use it today in the last of a series of four papers in the Proceedings of the Berlin Academy of Science in November 1915. Manuscript material such as the Einstein-Besso manuscript offers historians and philosophers of science a unique glimpse behind the scenes of Einstein at work.

Before I tell you more about the manuscript, let me tell you a little bit about myself and my interest in Einstein and the Einstein-Besso manuscript. My training is in theoretical physics and in history and philosophy of science. Before I came to the University of Minnesota in the fall of 2000, I worked for several years for the Einstein Papers Project, then located at Boston University (the project has since moved to Caltech in Pasadena, California). The goal of the Einstein Papers Project is to publish the definitive edition of Einstein's collected papers. This not only includes his published papers and books on physics and a range of other subjects (such as Zionism, pacifism, philosophy, obituaries, etc.) but also his correspondence, his lecture notes, his research notebooks, the occasional newspaper article or interview, etc. To date, eight of a projected twenty-nine volumes have appeared, covering Einstein's early years 1879–1902 (Vol. 1), his writings for the period 1902–1921 (Vols. 2, 3, 4, 6, 7), and his correspondence for the period 1902–1918 (Vols. 5, 8). I worked on material dealing with general relativity for Vols. 4, 7, and 8. The Einstein-Besso manuscript was published in Vol. 4.

As a historian and philosopher of science, I am interested in how Einstein came up with such a spectacular and strikingly novel theory as general relativity, which explains gravity in terms of the curvature of space-time. The analysis of documents such as the Einstein-Besso manuscript has helped me understand to some extent“how Einstein did it.” And this is precisely what I want to discuss in my segment of the course. Rather than talk about Einstein's MO in the abstract (which would be pure speculation) or look at what Einstein himself said about it reflecting on his work years later (which turns out to be highly misleading), I want examine the specific case of general relativity in the making. There is no better document to do so without extensive prior knowledge of the mathematics needed to formulate the theory than the Einstein-Besso manuscript. We can admire Einstein's magic from afar. But this manuscript actually allows us to take a peek behind the curtain of the wizard …

A Brief Characterization of the Einstein-Besso Manuscript and Its Importance

There are only two manuscripts still extant with research notes documenting Einstein's work toward the general theory of relativity. These two manuscripts are the so-called Zurich Notebook of late 1912/early 1913 and the Einstein-Besso manuscript, the bulk of which stems from June 1913. Of these two manuscripts, only the latter is in private hands. The Zurich Notebook is part of the Einstein Archives at Hebrew University in Jerusalem.

The published portion of the Einstein-Besso manuscript consists of about 52 loose pages, half of them in Einstein's hand, the other half in Besso's. There is no continuous numbering, which makes it hard to establish the exact order of the pages. The manuscript was brought to the attention of the editors of the Einstein Papers Project in 1988. Robert Schulmann, then the director of the project, was approached by Pierre Speziali, who told him that he had been given a manuscript by Vero Besso (1898–1971), the son of Michele, with whom he had become friendly in the course of editing the Einstein-Besso correspondence (Speziali 1972). Speziali made a copy of this manuscript available to the Einstein Papers Project, hoping that authentication of the manuscript by the Einstein editors would increase its value of this manuscript. Speziali wanted to sell the manuscript at auction to provide for his two daughters after his death. (I never got to meet Speziali, but he told Robert Schulmann that he liked my annotation of "his" manuscript. He died before Vol. 4 of the Einstein edition finally went to press.) The manuscript was eventually sold at auction by Christie's in 1996, fetching $360,000. The essay that you are about to read is adapted from the essay I wrote for the auction catalog. The current owner, a collector of scientific manuscripts in Chicago, has contacted Christie's to resell the manuscript at auction this fall along with some other Einstein material in his possession. In the summer of 1998, Robert Schulmann obtained an additional fourteen pages of the manuscript and there might well be more. This material is still in the possession of Besso's heirs. Negotations are underway with the Besso family to secure the necessary permissions to publish some of this additional material in a new book on Einstein's path to general relativity (Renn et al. forthcoming) . Here I will focus on the 52 pages that have appeared as Doc. 14 of Vol. 4 of the Einstein edition.

The aim of most of the calculations in the Einstein-Besso manuscript is to see whether an early version of the general theory of relativity, which Einstein had published in June 1913, can account for a tiny discrepancy between the observed motion of Mercury and the motion predicted on the basis of Newton's theory of gravity. This discrepancy is known as the anomalous advance of Mercury's perihelion. Exactly what that means will be explained below. The results of the calculations by Einstein and Besso were disappointing. The theory, as it stood, could only account for part of the discrepancy between observation and Newtonian theory. However, Einstein and Besso's efforts would not be in vain. The techniques developed in the manuscript for doing these calculations could be taken over virtually unchanged in November 1915 to compute the motion of Mercury predicted by the general theory of relativity in its final form. Einstein found that the final theory can account for the full effect left unexplained by Newtonian theory. As he later told a colleague, he was so excited about this result that it gave him heart palpitations (Pais 1982, p. 253).

The explanation of the anomalous advance of Mercury's perihelion would become one of the three classical tests of general relativity. Einstein found and published it in a time span of no more than a week. With the discovery of the Einstein-Besso manuscript, this impressive feat becomes more understandable: Einstein had done essentially the same calculation together with Besso two years earlier. The study of these earlier calculations has contributed to a fuller understanding of the 1915 perihelion paper.

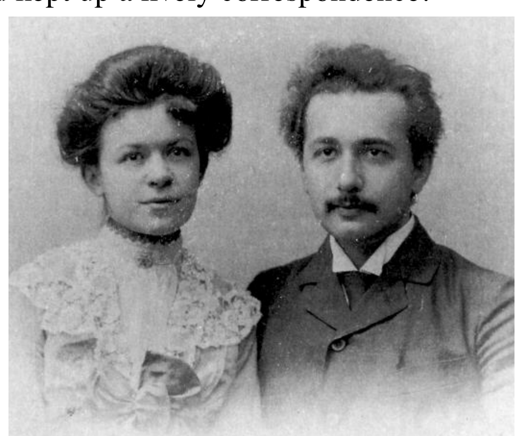
The Einstein-Besso manuscript is not only the key document for understanding the celebrated application of the general theory of relativity to the problem of Mercury's perihelion, but is also of great significance for the historical reconstruction of the genesis of the theory. In letters written shortly after his papers of November 1915, Einstein listed three reasons for abandoning the earlier version of his theory. The fact that the perihelion motion did not come out right was one of them. Another problem he mentioned was that the earlier theory was incompatible with the relativity of rotation. The Einstein-Besso manuscript contains an ingenious calculation, the whole purpose of which was to check whether the theory is compatible with this notion. Einstein was able to convince himself that the theory passed this test. However, he made some trivial errors in this calculation. More than two years later, in September 1915, he redid the calculation, this time without making any errors, and discovered, to his dismay, that the theory failed. In all likelihood, this discovery triggered the unraveling of the theory and led Einstein to return to ideas considered and rejected in the Zurich Notebook of some three years earlier. Using the same ideas, he was able to complete, in a little over a month, the general theory of relativity as we know it today.

Einstein and Besso: the Eagle and the Sparrow

Michele Besso and his wife
Anna Besso-Winteler

Einstein's historic 1905 paper "On the Electrodynamics of Moving Bodies," in which the theory now known as special relativity was announced, was unusual for a scientific paper in that it carried none of the usual references to the literature of theoretical physics. The only individual credited with any contribution to the 1905 paper was Michele Angelo Besso, whom Einstein thanked for "many useful suggestions." Besso, whom Einstein once characterized as a perpetual student, had studied mechanical engineering at the Zurich Polytechnic during the years Einstein was enrolled in the physics section. The two met at a musical evening in Zurich and remained lifelong friends. In 1904, on the recommendation of Einstein, Besso took a position at the Swiss Patent Office. Whenever they could, the two friends engaged in long discussions of physics and mathematics. Besso played a very important role as a "sounding board" for Einstein, and when Einstein moved to Zurich and later Berlin, the two men visited and kept up a lively correspondence.

When, shortly after taking up his position in Berlin in 1914, Einstein sent his wife Mileva and his sons Hans-Albert and Eduard back to Zurich, Besso and his wife took on the role of intermediary between the feuding partners as their marriage was dissolving. They even cared for the couple's two sons during Mileva's illness.



Albert Einstein and Mileva Einstein-Maric
(Wedding portrait, 1903)

In 1913, when Einstein and Besso collaborated on the calculations in the manuscript under discussion here, Besso was living in Gorizia, near Trieste. The manuscript shows that in this case Besso functioned as considerably more than just a sounding board. Although he left the hardest parts to Einstein, he did take responsibility for some important parts of the calculations. In later years, Besso described his scientific collaboration with Einstein with a charming simile: Einstein was an eagle, and he, Besso, a sparrow. Under the eagle's wing, the sparrow had been able to fly higher than on its own.

From Special Relativity to General Relativity

Einstein started on the path that would lead him to the general theory of relativity in late 1907. He was writing a review article about his 1905 special theory of relativity. The last section of this article was devoted to gravity. In it, Einstein argued that a satisfactory theory of gravity cannot be achieved within the framework of special relativity and that a generalization of that theory is needed. This was an extraordinary step to take at the time. Other researchers, such as the great mathematicians Henri Poincaré and Hermann Minkowski, felt that a perfectly adequate theory of gravitation could be constructed simply by modifying Newton's theory of gravitation somewhat to meet the demands of special relativity.

As Einstein was pondering the problem of gravitation, a thought occurred to him which he later described as "the happiest thought of my life" (Einstein 1920, [p. 21]). It had been known since Galileo that all bodies fall alike in a given gravitational field. This is the point of the famous, though most likely apocryphal, story of Galileo dropping bodies of different mass from the leaning tower of Pisa and watching them hit the ground simultaneously. Galileo's insight was incorporated but not explained in Newton's theory of gravity. It just happens to be the case that two very different quantities in Newton's theory have the same numerical value: the gravitational mass, a measure of a body's susceptibility to gravity, and the inertial mass, a measure of a body's resistance to acceleration.

Einstein found it unsatisfactory that the equality of inertial and gravitational mass was just a coincidence in Newton's theory. He felt that there had to be some deeper reason for it. He proposed to explain it by assuming that acceleration and gravity are essentially just one and the same thing. If that is true, it is not surprising that inertial mass, having to do with resistance to acceleration, and gravitational mass, having to do with susceptibility to gravity, are equal to one another. This idea is the core of what Einstein later dubbed the equivalence principle. Initially, Einstein formulated the equivalence principle as follows: the situation of an observer uniformly accelerating in the absence of a gravitational field is fully equivalent to the situation of an observer at rest in a homogeneous gravitational field. In particular, both observers will find that all free particles have the same acceleration with respect to them.

Formulated in this way, one can readily understand why Einstein felt that the equivalence principle could be used to extend the principle of relativity of his special theory of relativity of 1905, which only holds for uniform motion (i.e., motion with constant velocity), to arbitrary motion (i.e., accelerated motion, motion with changing velocity). An accelerated observer, Einstein reasoned, can always claim to be at rest in some gravitational field equivalent to his or her acceleration. In this manner, Einstein thought he could eliminate once and for all Newton's concept of absolute acceleration and formulate a theory in which all motion is relative: a general theory of relativity.

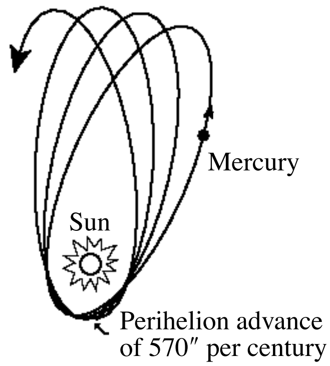
Unfortunately, the relation between acceleration and gravity turned out not to be quite as simple as Einstein initially thought. It took him till early 1918—more than two years after he had published the theory in the form in which it is still used today—to give a more accurate formulation of the relation between gravity and acceleration. He changed the definition of the equivalence principle accordingly, although he continued to use the old definition in popular expositions of his theory. The connection between gravity and acceleration or inertia is not that the two are always interchangeable. Rather it is that the effects we ascribe to gravity and the effects we ascribe to gravity are both produced by one and the same structure, a structure we now call the inertio-gravitational field, and which is represented in Einstein's theory by curved space-time. Observers in accelerated motion with respect to one another will disagree over how to split the effects of this inertio-gravitational field into effects of inertia and effects of gravitation. As Einstein explained in 1920 (Einstein 1920, [p. 20]), the situation is similar to the one he had arrived at in his work on special relativity. There he had replaced the electric field and the magnetic field by one electromagnetic field. Observers in relative motion with respect to one another will disagree over how to break down this one field into its electric and magnetic components. In working out the geometry of the new concepts of space and time of special relativity, the Göttingen mathematician Hermann Minkowski had done the same for space and time. He replaced space and time by one space-time. Observers in relative motion with respect to one another will disagree over how to break down a certain spatio-temporal distance into a spatial and a temporal distance component. The great novelty of general relativity—and perhaps its greatest lasting value—is that Einstein did the same thing for gravity and inertia. In 1918, he introdced a new version of the equivalence principle to bring out this central notion: there is only one field—a field we now call the inertio-gravitational field—that encompasses both the effects of gravity and of acceleration. The old equivalence principle is no more than a very special case of the new one. The main function of the old principle had been to guide Einstein in his search for his new theory of gravity.

Once the relation between gravity and acceleration has been identified correctly, we have to face up to the fact that the name "general theory of relativity" is a bit of a misnomer. Contrary to what Einstein had originally hoped, the theory does not fully eliminate absolute motion. Recall what absolute motion means in the context of Newton's theory. There it means that there is an absolute distinction between inertial motion—the motion of a body on which no external forces act—and non-inertial motion—the motion of a body subject to external forces. The formed motion will be represented by a straight line, the latter by a crooked line. A very similar distinction holds in Einstein's theory. There it is the distinction between inertio-gravitational motion—the motion of a body subject only to the influence of the curved space-time representing the effects of gravity and inertia—and non-inertio-gravitational motion—the motion of a body subject to influences over and above those of the inertio-gravitational field. The former motion is motion on straightest lines in curved space-time (think, for instance, of great circles on the surface of a sphere), the latter motion is on crooked lines in the curved space-time. These motions and these lines are called geodesic and non-geodesic, respectively.

All of these subtleties and many more, Einstein only learned along the way in his long and only partially successful quest for a general theory of relativity and a new theory of gravity from 1907 until about 1920. But—as we know from a letter to his friend Conrad Habicht of December 24, 1907—one of the goals that Einstein set himself early on, was to use his new theory of gravity, whatever it might turn ou to be, to explain the discrepancy between the observed motion of the perihelion of the planet Mercury and the motion predicted on the basis of Newtonian gravitational theory. This tiny discrepancy had been known for about half a century. Various explanations had been put forward during that time, none to Einstein's liking.

The Mercury Anomaly

Kepler's first law says that a planet moves on an ellipse with the sun in one of its focal points. Newtonian theory confirms this to a very good approximation. However, Newtonian theory also predicts that, because of the influence of other planets, these ellipses are not fixed in space but undergo a slow precession (see the figure at the top of the next page). One way to observe this precession is to follow the motion of the perihelion, the point where the planet is closest to the sun. For the perihelion of Mercury, Newtonian theory predicts (in a coordinate system at rest with respect to the sun) a secular advance of about 570″. "570″" means 570 seconds of arc, "secular" means per century. A second of arc is a very tiny unit. A circle has 360°, a degree has 60 minutes, and a minute has 60 seconds!



In 1859, the French astronomer Urbain Jean Joseph Le Verrier, after working on the problem for many years, pointed out that there was a discrepancy of about 38″ (i.e., about 1/100 of a degree per century!) between the value that Newtonian theory predicts for the secular motion of Mercury's perihelion and the value that was actually observed. Le Verrier suggested that perturbations coming from an additional planet, located between the sun and Mercury, were responsible for this discrepancy. In 1846, he had likewise predicted the existence of an additional planet to account for discrepancies between theory and observation in the case of Uranus. This planet, Neptune, was actually observed shortly afterwards, almost exactly where Le Verrier had predicted it to be. However, Vulcan, the planet to be made responsible for the discrepancy in the case of Mercury, was never found.

In 1895, the American astronomer Simon Newcomb published a new value for the anomalous secular advance of Mercury's perihelion, based on the latest observations. He arrived at about 41″. The modern value is about 43″. The best explanation for the anomaly that Newcomb could find was a suggestion made a year earlier by Asaph Hall that the gravitational force would not exactly fall off with the inverse square of the distance as in Newton's theory but slightly faster. Newton himself had already noted that any deviation from an exact inverse square law would produce a perihelion motion.

In the years following Newcomb's publication, several other explanations were put forward for the anomaly. The most popular one was offered by the German astronomer Hugo von Seeliger, who suggested the discrepancy was due, not to a planet between Mercury and the sun, but to bands of diffuse matter in that region.

In the decade following the publication of the special theory of relativity, various new theories of gravitation were suggested. For several of these theories—e.g., for theories by Henri Poincaré, Hermann Minkowski, Max Abraham, and Gunnar Nordström—it was checked explicitly whether they could account for the Mercury anomaly. None of them could. In Nordström's theory, the problem was even worse than in Newtonian theory, because the theory predicted a retrogression of about 7″ per century instead of the desired advance of a good 40″. All theories could, of course, easily be rendered compatible with observation as long as one was willing to accept Seeliger's hypothesis of intra-Mercurial matter.

It is against this background that the project documented in the Einstein-Besso manuscript should be seen. In June 1913, Michele Besso visited Einstein in Zurich. Together with his former classmate, the mathematician Marcel Grossmann, Einstein had just written a paper, which, as the title modestly announces, gives an *outline* of a generalized theory of relativity and a theory of gravitation. Einstein and Besso set themselves the task to find out whether this new theory could account for the Mercury anomaly.

The Einstein-Grossmann Theory

The Einstein-Grossmann theory—also known as the "Entwurf" ("outline") theory after the title of Einstein and Grossmann's paper—is, in fact, already very close to the version of general relativity published in November 1915 and constitutes an enormous advance over Einstein's first attempt at a generalized theory of relativity and theory of gravitation published in 1912. The crucial breakthrough had been that Einstein had recognized that the gravitational field—or, as we would now say, the inertio-gravitational field—should not be described by a variable speed of light as he had attempted in 1912, but by the so-called metric tensor field. The metric tensor is a mathematical object of 16 components, 10 of which independent, that characterizes the geometry of space and time.[1] In this way, gravity is no longer a force in space and time, but part of the fabric of space and time itself: gravity is part of the inertio-gravitational field. Einstein had turned to Grossmann for help with the difficult and unfamiliar mathematics needed to formulate a theory along these lines.

Any theory of the gravitational field can naturally be divided into two parts, a part describing how the gravitational field affects matter and a part describing how matter in turn generates gravitational fields. As far as the first part is concerned, the Einstein-Grossmann theory of June 1913 is identical to the general theory of relativity in its final form. The difference between the two theories concerns only the second part. The theories of 1913 and of 1915 have different *field equations* for the metric field, the field representing the gravitational field in these theories. Field equations are the equations that tell us what gravitational field is produced by a certain given matter distribution. During his collaboration with Grossmann, Einstein had actually considered field equations that are very close to the ones he would eventually settle on in November 1915. From a purely mathematical point of view, these equations were the natural candidates, but at the time Einstein convinced himself that from a physical point of view they would be unacceptable. Instead, he chose a set of equations, now known as the Einstein-Grossmann equations or the "Entwurf" field equations, that, although mathematically less elegant, he thought were more satisfactory from a physical point of view. This early struggle to find suitable field equations is documented in the Zurich Notebook of late 1912/early 1913, the only other extant research manuscript of this period.

Applying the Einstein-Grossmann Theory to the Problem of Mercury's Perihelion

The first step in calculating the perihelion advance predicted by the Einstein-Grossmann theory was to solve the "Entwurf" field equations to find the metric field produced by the sun. The second step would be to calculate the perihelion advance of a planet moving in this field.

In order to solve the field equations, Einstein came up with an ingenious iterative approximation procedure. First, he calculated the metric field of a point mass (representing the sun) using the field equations in a first-order approximation. He then substituted the result of this calculation

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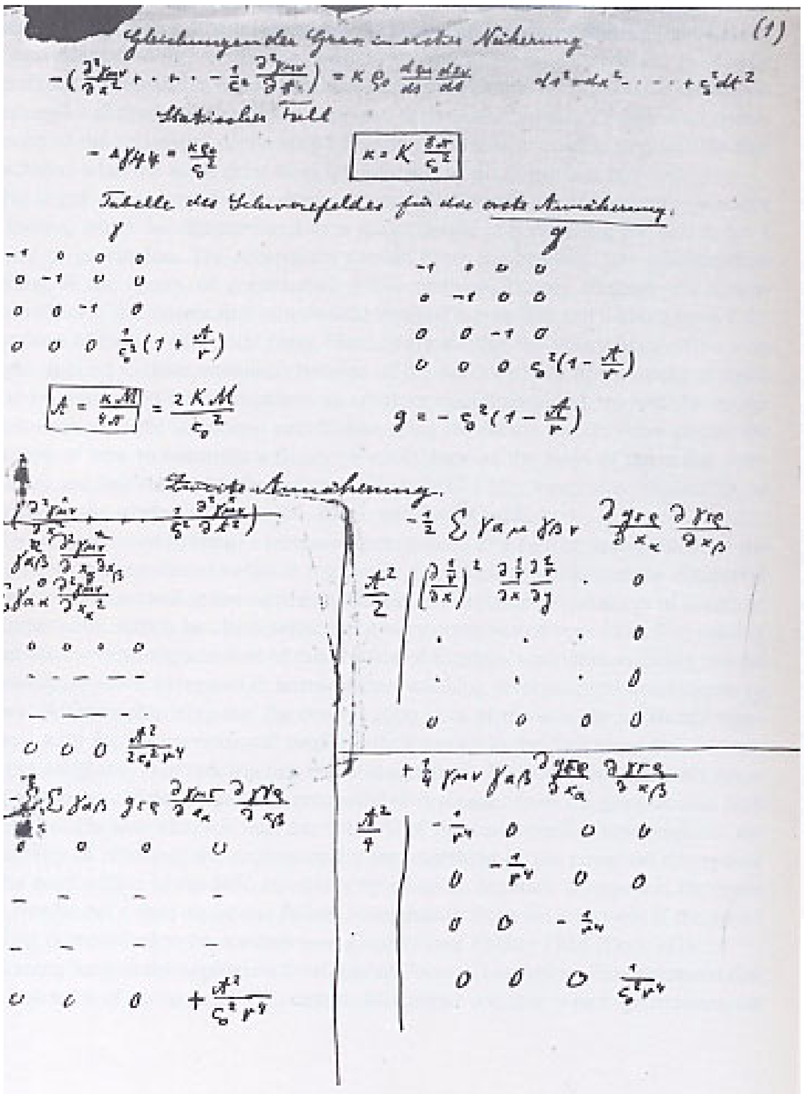
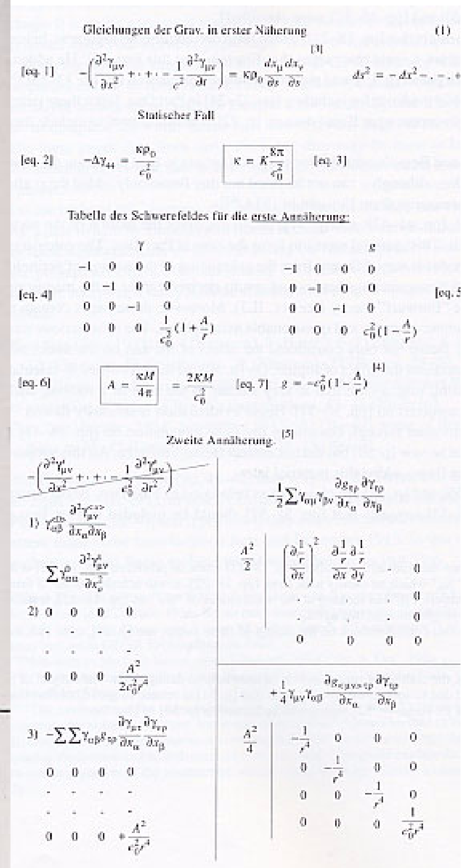
1. The following analogy should suffice to understand the notion of a *metric tensor field* well enough to understand this essay. Consider a map of the earth (which is essentially a way of coordinatizing the globe). We cannot simply take distances on the map (the coordinate distances) to represent distances on the globe (the actual or proper distances). For instance, a horizontal line segment of two inches on the map near the equator will correspond to a larger distance on the globe than a horizontal line segment of two inches on the map near the poles. For every point on the map, we need to specify a set of numbers with which we have to multiply distances on the map in the vicinity of that point (*coordinate distances*) to convert them to actual distances (*proper distances*). (It will be clear that we need more than one number because the conversion for north-south distances will be different from the conversion for east-west distances.) The numbers in such a set are called the *components of the metric tensor* at that point. The metric tensor *field* is the collection of all such sets of numbers for all points on the map. The same thing we do here with 2-dim. space (representing the 2-dim. curved surface of the earth on a 2-dim. Euclidean plane together with a specification of the metric field to do all conversion from coordinate distances to proper distances) we can do with 4-dim. space-time as well. There will now also be a number (the temporal component of the metric) by which we have to multiply coordinate time differences to convert them to proper time differences.

back into the field equations, now in a more accurate second-order approximation, and solved these equations to obtain more accurate expressions for the metric field of the point mass. This two-step procedure nicely illustrates the fundamental physical complication that underlies the complexity of the equations Einstein had to work with: due to the equivalence of mass and energy, the gravitational field, by virtue of carrying a certain amount of energy, acts as its own source. So, in Einstein's second-order approximation, both the energy of the point mass and the energy of the metric field has to be taken into account.

Finding a sufficiently accurate expression for the metric field of the sun takes up the first few pages of the Einstein-Besso manuscript (pp. 1–7 of the manuscript as presented in Vol. 4 of the Einstein edition). These pages are all in Einstein's hand, with just a few corrections of minor slips in Besso's hand. Below is the first of these pages, both in facsimile and in transcription:





Gleichungen der Grav. in erster Näherung (1)

[eq. 1] $-\left(\frac{\partial^2\gamma_{\mu\nu}}{\partial x^2}+\cdot+\cdot-\frac{1}{c^2}\frac{\partial^2\gamma_{\mu\nu}}{\partial t}\right)=\kappa\rho_0\frac{dx_\mu}{ds}\frac{dx_\nu}{ds}$ [3] $ds^2=-dx^2-\cdot-\cdot+$

Statischer Fall

[eq. 2] $-\Delta\gamma_{44}=\frac{\kappa\rho_0}{c_0^2}$ $\boxed{\kappa=K\frac{8\pi}{c_0^2}}$ [eq. 3]

Tabelle des Schwerefeldes für die erste Annäherung:

[eq. 4] γ:
$$\begin{matrix}-1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&\frac{1}{c_0^2}(1+\frac{A}{r})\end{matrix}$$

g: [eq. 5]
$$\begin{matrix}-1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&c_0^2(1-\frac{A}{r})\end{matrix}$$

[eq. 6] $\boxed{A=\frac{\kappa M}{4\pi}}=\frac{2KM}{c_0^2}$ [eq. 7] $g=-c_0^2(1-\frac{A}{r})$ [4]

Zweite Annäherung. [5]

$-\left(\frac{\partial^2\gamma'_{\mu\nu}}{\partial x^2}+\cdot+\cdot-\frac{1}{c_0^2}\frac{\partial^2\gamma'_{\mu\nu}}{\partial t^2}\right)$

1) $\gamma^{\langle 1\rangle}_{\alpha\beta}\frac{\partial^2\gamma^{\langle 1\rangle}_{\mu\nu}}{\partial x_\alpha\partial x_\beta}$

$\sum\gamma^0_{\alpha\alpha}\frac{\partial^2\gamma^\delta_{\mu\nu}}{\partial x_\alpha^2}$

2)
$$\begin{matrix}0&0&0&0\\\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot\\0&0&0&\frac{A^2}{2c_0^2r^2}\end{matrix}$$

3) $-\sum\sum\gamma_{\alpha\beta}g_{\tau\rho}\frac{\partial\gamma_{\mu\tau}}{\partial x_\alpha}\frac{\partial\gamma_{\nu\rho}}{\partial x_\beta}$
$$\begin{matrix}0&0&0&0\\\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot\\0&0&0&+\frac{A^2}{c_0^2r^4}\end{matrix}$$

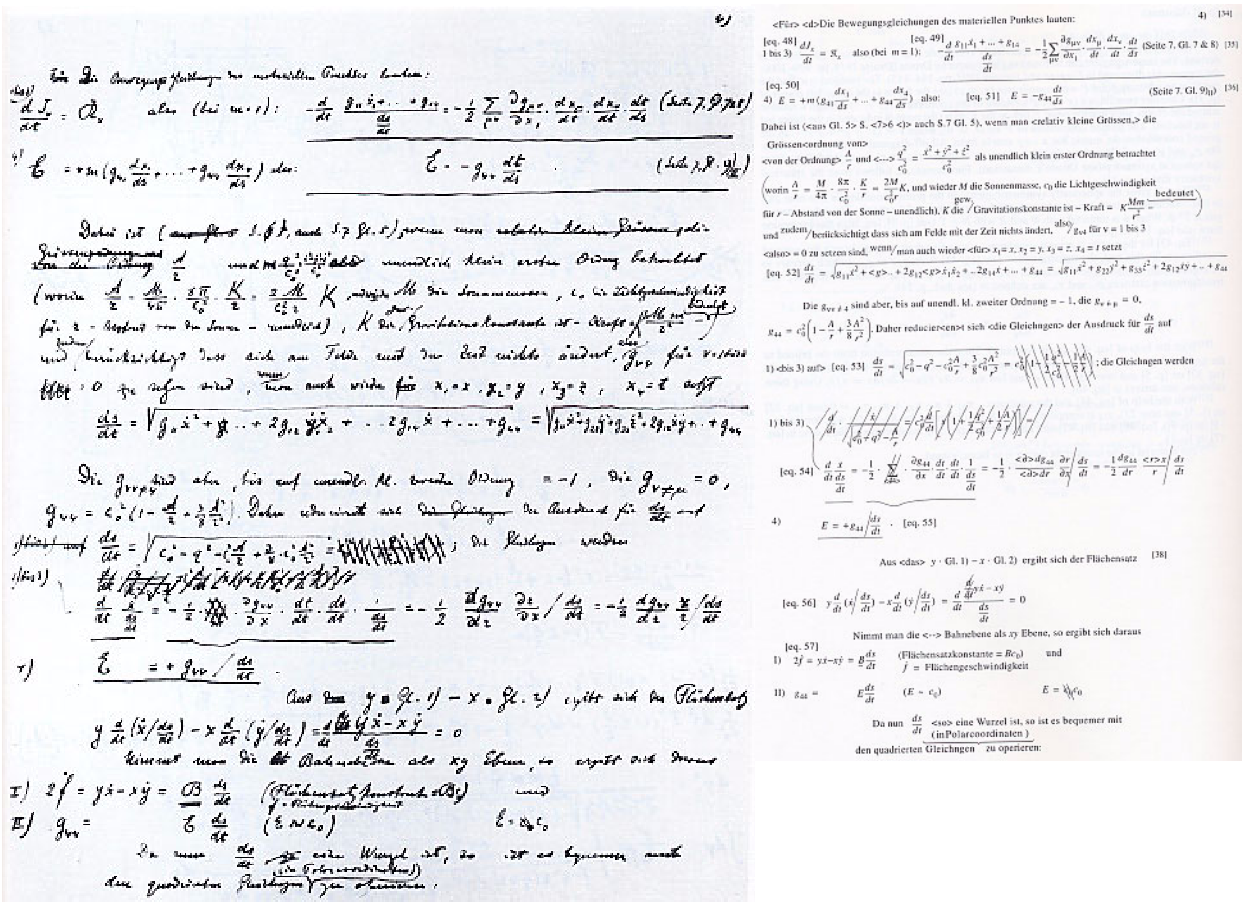
$-\frac{1}{2}\sum\gamma_{\alpha\mu}\gamma_{\beta\nu}\frac{\partial g_{\tau\rho}}{\partial x_\alpha}\frac{\partial\gamma_{\tau\rho}}{\partial x_\beta}$

$$\frac{A^2}{2}\left|\begin{matrix}\left(\frac{\partial\frac{1}{r}}{\partial x}\right)^2&\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial\frac{1}{r}}{\partial y}&\cdot&0\\\cdot&\cdot&\cdot&0\\\cdot&\cdot&\cdot&0\\0&0&0&0\end{matrix}\right.$$

$+\frac{1}{4}\gamma_{\mu\nu}\gamma_{\alpha\beta}\frac{\partial g_{\tau\rho}}{\partial x_\alpha}\frac{\partial\gamma_{\tau\rho}}{\partial x_\beta}$

$$\frac{A^2}{4}\left|\begin{matrix}-\frac{1}{r^4}&0&0&0\\0&-\frac{1}{r^4}&0&0\\0&0&-\frac{1}{r^4}&0\\0&0&0&\frac{1}{c_0^2r^4}\end{matrix}\right.$$

Besso takes over on the next couple of pages (pp. 8–9), deriving an equation for the perihelion motion of a planet in the metric field of the sun. The first of these pages is shown at the top of the next page, again both in facsimile and in transcription. Notice that this page is not nearly as "clean" as Einstein's. Einstein is clearly much more comfortable with the calculations than Besso is. Besso's insecurity is reflected in the many deletions he makes and also in the fact that he uses a lot more explanatory prose than Einstein does. On the next two pages (pp. 10–11), Einstein takes over again and finds an expression for the perihelion advance of a planet in the field of the sun. Besso then rewrites this equation, making sure that it only contains quantities for which numerical values would be readily available, some astronomical data pertaining to the sun and Mercury along with some constants of nature (p. 14).

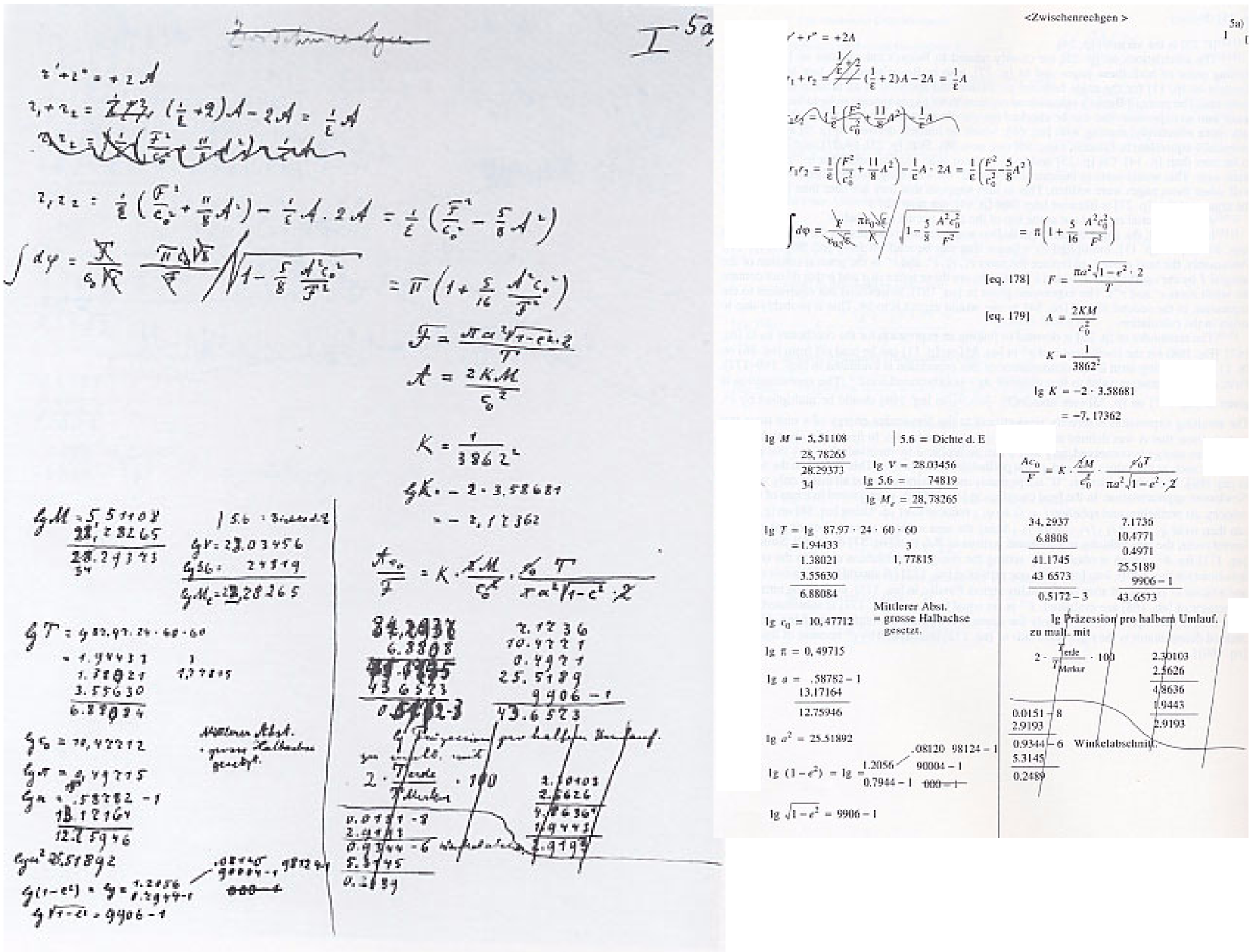


It is only later in the manuscript (on p. 26 and p. 28) that actual numbers are inserted in this expression, and that the result is converted from the units used in the calculation to those used in observations. The end result is given by Einstein on p. 28: "1821″ = 30′ unabhängig geprüft" (independently checked). This result is disastrous. The hope was that the sun, which in Newton's theory produces no perihelion motion at all, would, in the Einstein-Grossmann theory, produce the 43″ of the observed 570″ that cannot be attributed to other planets. Instead, it looks as if the sun alone produces a perihelion motion of more than three times the size of the *total* perihelion motion that is observed!

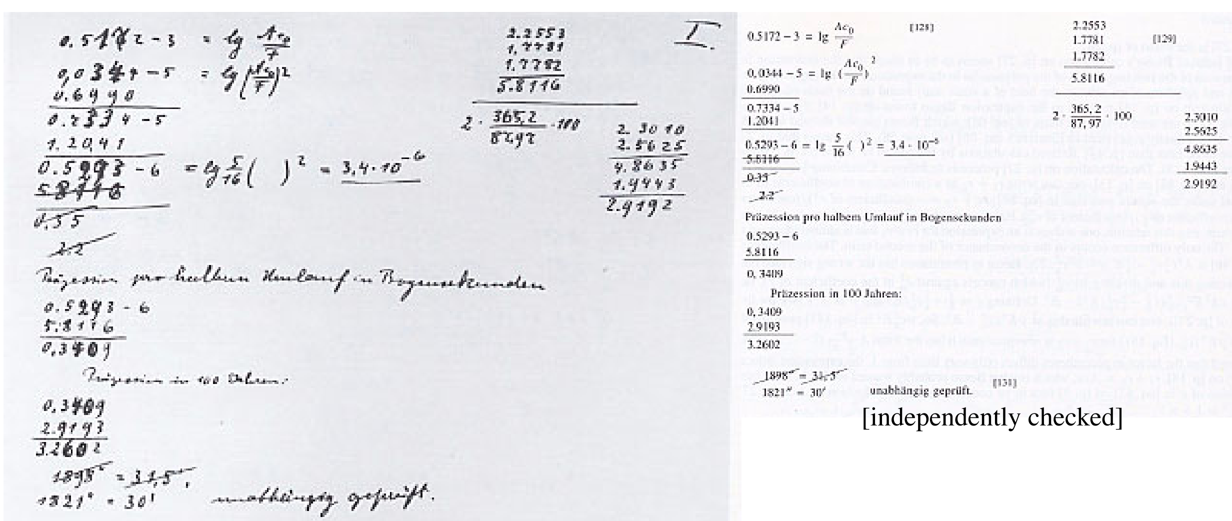
Fortunately, there is a mistake in the numerical calculation. The value for the mass of the sun is off by a factor of 10. Since the perihelion motion is proportional to the square of this quantity, the final result is off by a factor 100. So, the Einstein-Grossmann theory actually predicts that the sun produces an advance of Mercury's perihelion of about 18″ per century. Even though neither Einstein nor Besso corrected the bizarre result of about 1800″ on p. 28, it is clear that they discovered their mistake. On p. 35, in the context of another numerical calculation, there is a correction in Besso's hand of the erroneous value for the mass of the sun that Einstein had used. And on

p. 30, Einstein himself, replaced the value $3.4 \cdot 10^{-6}$ which, when converted, gives the bizarre $1800''$ of p. 28, by $3.4 \cdot 10^{-8}$, which gives the correct $18''$.

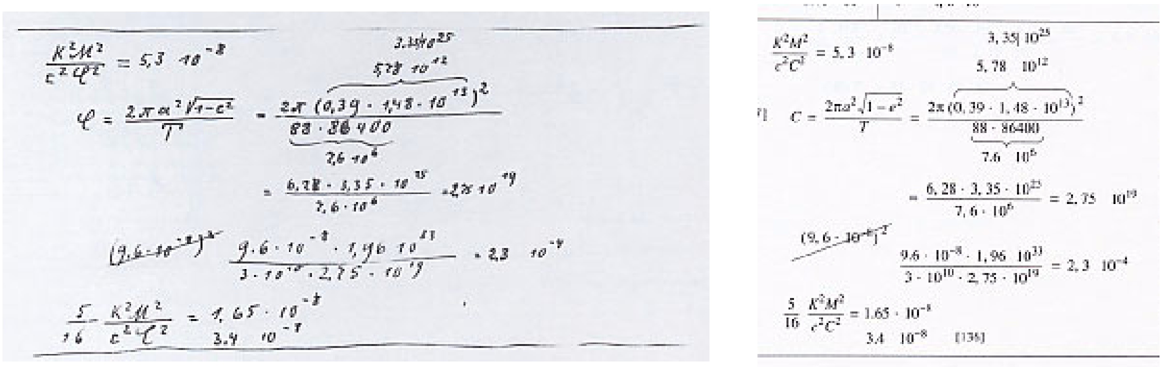
Below and on the next page, these four pages with numerical results (pp. 26, 28, 30, and 35) are reproduced.



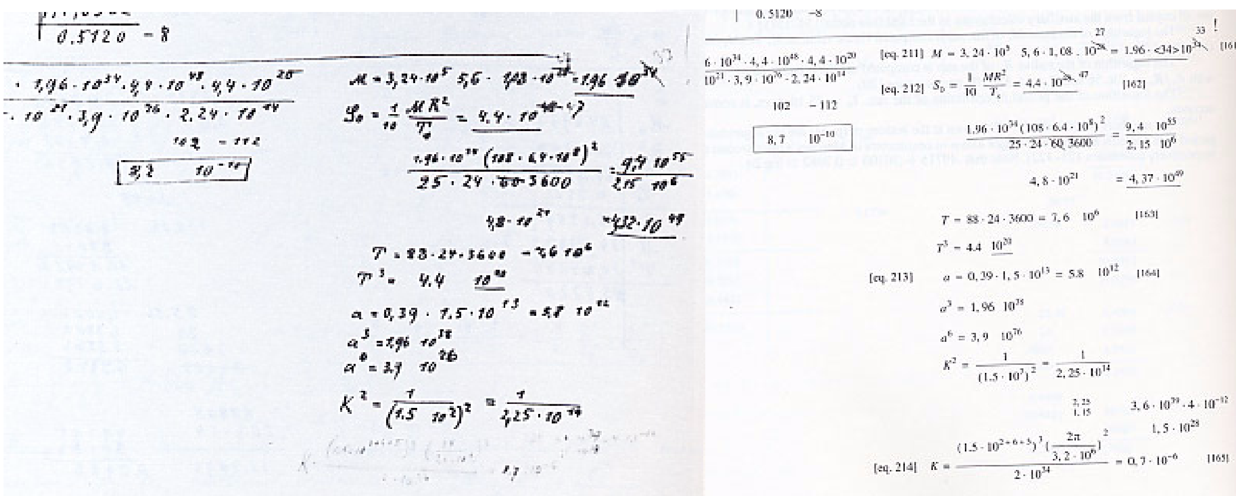
p. 26: Using logarithms to evaluate the perihelion advance per half a revolution



p. 28: Converting the result from fractions of π per half a revolution to seconds of arc per century



p. 30: Einstein realizes that the perihelion advance should be $3.4 \cdot 10^{-8}$ fractions of π per half a revolution



p. 35: Besso realizes that the value for the volume of the earth in the formula $M_{\text{Sun}} = \left(\frac{M_{\text{Sun}}}{M_{\text{Earth}}}\right)\rho_{\text{Earth}}V_{\text{Earth}}$ is a factor 10 too big.

Of course, even the corrected value of 18″ was disappointing. One would still need Seeliger's hypothesis of intra-Mercurial matter to account for the remaining part of the 43″ discrepancy. Einstein and Besso therefore considered other effects in the Einstein-Grossmann theory that might contribute to the perihelion motion. In particular, they considered the effect of the rotation of the sun. In Newtonian theory, the sun's rotation would not produce any perihelion motion at all, but in the Einstein-Grossmann theory (as in general relativity in its final form) it does have a small effect. On p. 35, the final result of Einstein and Besso's calculations for this effect is given as $8.7 \cdot 10^{-10}$, which amounts to about 0.1″ per century, which is negligible. In fact, Einstein once again used the wrong value for the mass of the sun, so the result should really be 0.001″. Einstein probably was pleased to find that the effect is small, because the effect of the rotation of the sun is not an advance but a retrogression of the perihelion.

The Fate of the Einstein-Besso Manuscript

Before they could finish their joint project in June 1913, Besso had to leave Zurich and go back to Gorizia where he lived at the time. The additional pages found in 1998 make it clear that Besso visited Einstein again in late August 1913. At that time, they worked on the project some more and added a few more pages. The manuscript stayed behind with Einstein in Zurich. Early in 1914, Einstein sent it to Besso, urging his friend to finish their project.

Besso added more material some of which can be found on pp. 45–53 and on pp. 41–42 of the published portion of the manuscript. He investigated three other possible contributions to the motion of Mercury's perihelion: the effect of the sun's rotation on the motion of the *nodes* of the planet (i.e., the points where the orbit intersects the ecliptic); a similar effect due to the motion of Jupiter; and, finally, the effect of solar pressure. There are a number of mathematical and arithmetical errors in these calculations, which in some cases led Besso to seriously overestimate the magnitude of these effects. But even with these errors, none of the effects led to contributions to the perihelion motion of the desired magnitude and/or sign.

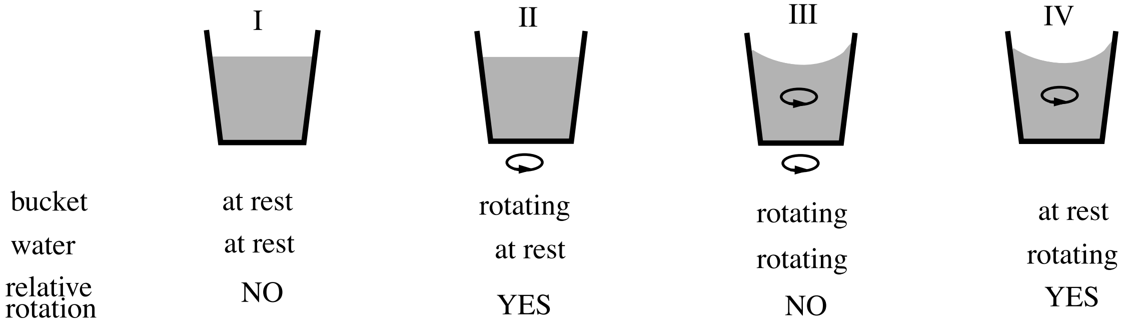
Besso also calculated the perihelion motion predicted by the Nordström theory using some of the same techniques he and Einstein had used in the context of the Einstein-Grossmann theory. Despite the fact that this calculation also contains some (minor) errors, it shows that Besso had gotten quite proficient at performing this type of calculation. Still, Besso did not pursue the topic any further. For the rest of his life, however, he held on to the notes of this joint work with the friend he so ardently admired. It is thanks this admiration that the manuscript survives at all. Had it remained in Einstein's possession, it would almost certainly have been discarded. After all, it consists entirely of scratch pad calculations. It would not have been worth saving for Einstein. But it is extremely valuable to historians of relativity, since it gives us a glimpse of Einstein at work at the height of his creative powers.

The Einstein-Grossmann Theory and the Problem of Rotation

Einstein did not give up the Einstein-Grossmann theory once he had established that it could not fully explain the Mercury anomaly. He continued to work on the theory and never even mentioned the disappointing result of his work with Besso in print. So Einstein did not do what the influential philosopher Sir Karl Popper claimed all good scientists do: once they have found an empirical refutation of their theory, they abandon that theory and go back to the drawing board. If scientist religiously stuck to that principle, very little progress would be made. A certain tenacity in working on a theory even in the face of empirical problem is needed to give that theory a fighting chance. The thing to watch out for is that such tenacity does not turn into the stubbornness of refusing to let go of a beloved theory after it has become clear that no amount of fixing it up is going to turn it into a viable theory.

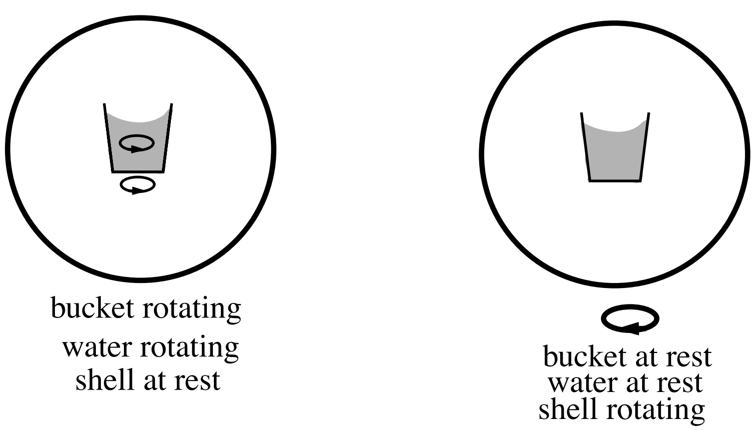
In late 1914, the Dutch physicist Johannes Droste independently found and published the basic result of the Einstein-Besso manuscript, viz. that the Einstein-Grossmann theory could account for only $18''$ of the Mercury anomaly. This was not seen as particularly damning to the Einstein-Grossmann theory in the scientific community. Einstein, at this point, firmly believed in the correctness of his theory. In October 1914, he had presented what looks like a definitive presentation of the theory. What eventually prompted Einstein to give up the Einstein-Grossmann theory had nothing to do with the Mercury anomaly. It did have to do, however, with another calculation preserved in the Einstein-Besso manuscript.

One of the virtues of the Einstein-Grossmann theory emphasized in Einstein's lengthy exposition of the theory of October 1914 is that it does away with the notion of absolute rotation. Newton had illustrated the absolute character of rotation in the famous thought experiment of the rotating bucket, which is illustrated below.



A bucket filled with water, initially at rest (stage I), is set spinning. At first the water remains at rest and the water surface stays flat (stage II). After a while the water starts "catching up" with the rotation of the bucket and the water surface becomes concave. At some point the water will be spinning just as fast as the bucket (stage III). Now grab the bucket. The bucket will instantaneously be brought to rest while the water continues to spin. The water surface maintains its concave shape. Now notice that in stages I and III there is no relative rotation of the water with respect to the bucket, whereas in stages II and IV there is. If the shape of the water were determined by the relative rotation of the water with respect to the bucket, the shapes in stages I and III and the shapes in stages II and IV should be the same. However, they clearly are not. Even though the relative rotation of the water and the bucket is the same in stages I and III (and in stages II and IV), the surface is flat in one case and concave in the other. This proves, as Newton argued, that the shape of the surface cannot be due to the relative rotation of the water and the bucket. The shape of the surface is determined not by the relative but by the absolute rotation of the water, the rotation with respect to Newton's absolute space.

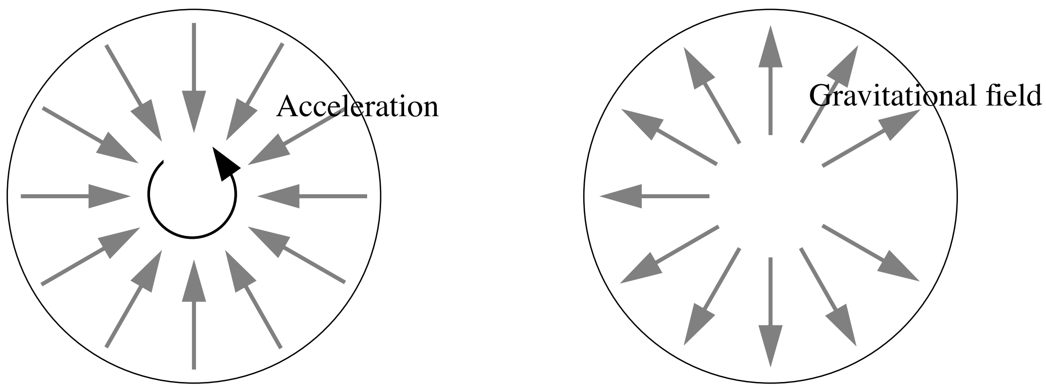
In the late 19th century, Ernst Mach suggested that the effect might still be explained in terms of relative rotation, not of the water with respect to the bucket, but of the water and the bucket with respect to the rest of the universe. In order for this explanation to work, it has to be the case that the theory predicts the same effect for the case where the water-filled bucket is rotating and the distant stars are at rest as it does for the case where the bucket is at rest and the distant stars are rotating (see the figure below).



Unfortunately, Newton's theory, while predicting the observed effect in the former case, predicts no effect at all in the latter. The Einstein-Grossmann theory, however, Einstein suggested, does predict the same effect in the two cases and hence allows one to relativize Newton's absolute rotation along Machian lines.

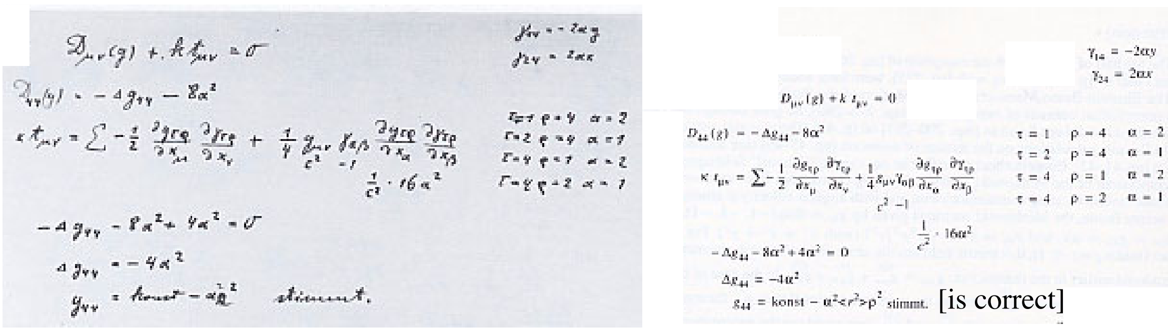
There are two calculations in the Einstein-Besso manuscript pertaining to these claims about rotation, one on pp. 36–37, the other one on pp. 41–42. On pp. 36–37, Einstein calculated, in a first-order approximation, the metric field that a rotating shell would produce near its center. The shell obviously represented the distant stars. As Einstein reported to Mach in June 1913, the Einstein-Grossmann theory, unlike Newtonian theory, does predict a small effect of the type needed for a Machian account of Newton's bucket experiment.

On pp. 41–42, Einstein arrived at an even more encouraging result. According to the early version of the equivalence principle, rotation should be equivalent to some gravitational field. This is illustrated in the figure below. Standing on a rotating merry-go-round your velocity constantly changes direction, which means that you are experiencing a centripetal acceleration. This feels as if there is a force—known as the centrifugal force—that is trying to throw you off the merry-go-round. According to the equivalence principle, that situation should be fully equivalent to one in which you are standing on a merry-go-round at rest in a peculiar centrifugal gravitational field.



One can easily calculate the metric field describing space and time from the point of view of an observer in uniform rotation. The early version of the equivalence principle requires that this metric field can also be interpreted as a gravitational field. This means that it should be a solution of the field equations of the theory. To check this, Einstein used the same approximation procedure he had used to find the field of the sun for the perihelion calculations. In first-order approximation, it is easily verified that the metric field for the rotating observer is indeed a solution of the field equations of the Einstein-Grossmann theory. Moreover, this first-order metric field has the same form as the first-order metric field for the case of the rotating shell found on pp. 36–37, which fits nicely with the idea that this metric field can be interpreted as the field produced by the distant stars rotating with respect to the observer. Einstein then substituted this first-order field into the field equations in a second-order approximation and checked whether the metric field of the rotating observer is also a solution to the equations at this further level of approximation. He concluded that it is. Next to the final result of his calculation on p. 41, he wrote: "stimmt" ("is correct"). The relevant passage is reproduced at the top of the next page.

Unfortunately, Einstein made some trivial mistake in this calculation. The metric field describing space and time for a rotating observer is not a solution of the field equations of the Einstein-Grossmann theory. Einstein had been so convinced that it was that he had not been careful enough in checking this important aspect of the theory.



p. 41: Einstein misses the problem of rotation

How Rotation Brought Down the Einstein-Grossmann Theory and How Mercury's perihelion Confirmed Its Successor

In September 1915, Einstein for some reason redid the calculation of pp. 41–42 of the Einstein-Besso manuscript and discovered the error he had made over two years earlier. This must have come as a severe blow. This was a problem far worse than the theory's prediction of the perihelion motion of Mercury being a few seconds of arc off. This problem went straight to the core of Einstein's theory, the idea that gravity and acceleration are essentially one and the same thing. Tenacity would turn into stubbornness if Einstein had continued to hold on to the theory in the face of this disaster. As we saw earlier, when Einstein was ready to abandon his own 1905 theory in 1907 just when others were warming up to it, Einstein was anything but stubborn. Shortly after discovering the problem with rotation for the field equations of the Einstein-Grossmann theory, Einstein gave up these field equations.

He then returned to the other mathematically more elegant candidates he had considered but rejected nearly three years earlier in his work with Grossmann preserved in the Zurich notebook. Einstein had learned a good deal about theories based on a metric tensor field in the meantime and as a result he was able to quickly overcome his objections to those equations of 1912/1913. On November 4, 1915, he presented a paper to the Berlin Academy officially retracting the Einstein-Grossmann equations and replacing them with new ones. On November 11, a short addendum to this paper followed, once again changing his field equations. A week later, on November 18, Einstein presented the paper containing his celebrated explanation of the perihelion motion of Mercury on the basis of this new theory. Another week later he changed the field equations once more. These are the equations still used today. This last change did not affect the result for the perihelion of Mercury.

Besso is not acknowledged in Einstein's paper on the perihelion problem. Apparently, Besso's help with this technical problem had not been as valuable to Einstein as his role as sounding board that had earned Besso the famous acknowledgment in the special relativity paper of 1905. Still, an acknowledgment would have been appropriate. After all, what Einstein had done that week in November, was simply to redo the calculation he had done with Besso in June 1913, using his new field equations instead of the Einstein-Grossmann equations. It is not hard to imagine Einstein's excitement when he inserted the numbers for Mercury into the new expression he found and the result was 43″, in excellent agreement with observation. Fortunately, this result is not affected by a final modification of the field equation, which Einstein introduced in one last communication to the Prussian Academy on November 25.

Epilogue

On November 19, the famous Göttingen mathematician David Hilbert sent Einstein a postcard congratulating him on his success in explaining the Mercury anomaly. He expressed his admiration for the speed with which Einstein had done the necessary calculations. Einstein did not let on that this was basically because he had already done the same calculation two years earlier with a less happy result. Little did he know at that point that his friend and admirer Michele Besso would preserve these earlier calculations for posterity.

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