Taming the Gamma Function

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The Gamma (Factorial) function and its many applications in mathematics, engineering and physics requires no explanation. The Gamma integral is hard to evaluate by hand, but Gamma function calculators are now commonplace on computers and handheld calculators. Nevertheless, here we present a simple empirical expression for the factorial $n!$ that is nearly exact for the range $1 \leq n < \infty$ as a suitable replacement for Stirling's formula and other approximations. Because $n!$ blows up quickly as n gets large, we restrict our approach to computing $\ln(n!)$.

The Gamma function is defined as

$$\Gamma(n+1) = \int_0^\infty x^n e^{-x} dx$$

which, for integer values of n, gives $n!$. But oddly enough it also provides values for positive, non-integer n as well. The $n+1$ input seems strange, but it can easily be rearranged to provide $n!$ directly.

Probably the most famous approximation for calculating $n!$ is Stirling's formula, which is

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

which provides decent values for $n!$ when the argument n is very large. There are other approximation formulas available for small n, but none of them is very accurate.

By curve fitting known values of $\ln(n!)$ over the range $1 \leq n < \infty$, we can derive a highly accurate formula suitable for general use. It is

$$\ln(n!) = \left(n + \frac{1}{2}\right)\ln(n + 0.1194) - n + 0.7995 + \frac{0.0314}{n} \tag{1}$$

which typically gives results accurate to four decimal places over the given range. Its accuracy increases as n gets truly large, approaching that given by the Gamma function itself.

Note that as n grows, the expression approximates $\ln(n!) = n\ln(n) - n$, which is the extremum of the Gamma function itself. For example, the above expression for $\ln(1,234,567!)$ gives $16,081,762.690$, which is essentially the same as the true value $16,081,762.685$.

Here are several examples using (1), with the actual values in bold:

$$\ln(1!) = 0.000089, \text{or } 1! = 1.000089;\ \ \mathbf{0};\ \mathbf{1}$$

$$\ln(5!) = 4.787484, \text{or } 5! = 119.999071;\ \ \mathbf{4.787492};\ \mathbf{120}$$

$$\ln(13.1352!) = 22.904747, \text{or } 13.1352! = 8.859417 \times 10^9;\ \ \mathbf{22.904752};\ \mathbf{8.859470 \times 10^9}$$

$$\ln(200!) = 863.231952, \text{or } 200! = 7.886301 \times 10^{374};\ \ \mathbf{863.231987};\ \mathbf{7.886578 \times 10^{374}}$$

Note that while the empirical values for $\ln(n!)$ get more accurate as n gets larger, the exponentiation process introduces unavoidable decimal overflow errors for $n!$ itself.

Reference

I've used Polysoftware's PSI-Plot for several years now for basic statistical work, especially curve fitting. It compares favorably with much more costly mathematical software, although it only runs on Windows.
https://www.polysoftware.com/plot.htm