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From Hermann Weyl to Yang and Mills to Quantum Chromodynamics

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This is a personal view of [1] the developments from the invention of the concept of gauge invariance to our present understanding that it provides the fundamental principle for the construction of theories of forces between the basic blocs of matter. This journey was full of twists and turns and marked by fascinating moments. It is these aspects of the development of gauge theories that I will concentrate on.

Although Yang-Mills theories provide the basic framework for both strong and electroweak interactions, my contribution concerns almost exclusively the former only.

1. The beginning of all: Hermann Weyl and the concept of gauge invariance

The concept of local gauge invariance goes back to the attempt of Hermann Weyl in 1918 to generalize Riemannian geometry by discarding one of its basic assumptions. His attempt, as well as the next one of 1929, are discussed in detail in an excellent essay [4]. For Weyl the fact that in Riemannian geometry the metric allows the magnitudes of vectors to be compared not only at the same point, but at any arbitrary distant points represented an element of geometry "at a distance", which he disliked. According to him

"A true infinitesimal geometry should, however, recognize only a principle for transfering the magnitude of a vector to an infinitesimally close point and then, on transfer to an arbitrary distant point the integrability of the magnitude of a vector is no more to be expected than the integrability of its directions."

Once this "inconsistency" is removed, Weyl observed that

"... there appears a geometry that, surprisingly, when applied to the world, explains not only gravitational phenomena, but also the electrical. According to the resultant theory both spring from the same source, indeed in general one cannot separate gravitation from electromagnetism in a unique manner."

Though mathematically beautiful, Weyl's theory did not describe reality and he had to abandon it. In 1929, shortly after the formulation of QED, Weyl tried again, this time relating electromagnetism to matter field. In the Abstract of his great paper [5] he states

"The Dirac field equations for ψ together with the Maxwell equations for the four potentials f_p of the electromagnetic field have an invariance property ....the equations remain invariant when one makes simultaneous substitutions $\psi \to e^{i\lambda}\psi, f_p \to f_p - \partial\lambda/\partial x_p$. It seems

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[1] There are many excellent articles discussing various aspects of the development of Yang-Mills theories [1–3]. The contribution of Weyl toward the concept of gauge invariance is discussed in [4].

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to me that this new principle of gauge invariance, which follows not from speculation but from experiment, tells us that the electromagnetic field is a necessary accompanying phenomenon not of gravitation, but of material wave field represented by ψ."

This time he got it right and the concept of gauge invariance was born. Weyl's route to gauge invariance illustrates the fact that although as demanded by Dirac *Physical laws must have mathematical beauty*, the converse is not true: *mathematical beauty does not imply physical relevance*! It is ultimately the experiment which decides the latter.

2. Step in extra dimensions: Oscar Klein and his Theory of Everything

Another attempt to derive all then known forces starting from gravity was carried out by Oscar Klein in his remarkable paper presented at the Warsaw conference "New Theories in Physics" in August 1938. Klein's contribution is explained in detail by David Gross in [6] and I will therefore merely recall the essence of Klein's work.

Klein's contribution can be regarded as the first attempt at constructing the Theory of Everything. In order to describe photon and two charged intermediaries predicted in 1935 by Yukawa, Klein extended the framework he developed earlier for deriving electromagnetism in four dimensional world from gravitation in five dimensional space-time. His immediate motivation was the discovery of what was then called "mesoton". It took some time to realize that this particle, now called muon, cannot be the particle mediating strong interactions, but Klein had not made a difference between strong and weak interactions and his mesoton thus mediated not only in strong forces between protons and neutrons, as envisaged by Yukawa, but also weak decays, like the neutron β-decay. It was known to people like Yukawa, Tamm and Ivanenko that this assumption was in conflict with experimental evidence but Klein had apparently not been aware of their arguments.

The theory Weyl constructed thus included the photon and two charged intermediate vector mesons with correct couplings, characteristic of the SU(2) nonabelian gauge theory, between all of them. Nevertheless, it was not a genuine SU(2) gauge theory, because the coupling of this triplet to nucleon and lepton doublets was not of the form required in SU(2) gauge theory. As we know today, one needs more complicated group to achieve that. Moreover as Klein did not aim at constructing SU(2) gauge theory, but wanted to described the observed mesotons, he felt free to add by hand the corresponding mass term. Klein thus came close to nonebalian gauge theory, but not quite.

3. Premature burial

At about the same time Yang and Mills were working on the generalization of gauge invariance, Landau and Pomeranchuk had been attempting to give the renormalization procedure developed at the end of forties to tame the ultraviolet divergencies in QED a good physical sense. Their attempt [8] had failed which led them to a far-reaching conclusion: quantum electrodynamics makes, strictly speaking, no sense. Though not generally accepted in this extreme form, the conclusion of Landau and Pomeranchuk was shared by a number of founding fathers of quantum theory, among them Dirac. As late as 1974 this is what he said about the renormalized QED [9]:

"Hence most physicists are very satisfied with the situation. They say: "Quantum electrodynamics is a good theory, and we do not have to worry about it any more." I must

say that I am very dissatisfied with the situation, because this so-called "good theory" does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics.

Of course, the proper inference from this work is that the basic equations are not right $\cdots$. *There must be some drastic change introduced into them so that no infinities occur in the theory at all and so that we can carry out the solution of the equations sensibly, according to ordinary rules."*

Dirac was right as far as QED is concerned, but wrong in general. The importance of Yang-Mills theories lies, among other things, in the fact that for (some of) these theories the renormalization procedure as formulated by Landau leads to interacting quantum field theories well-defined down to infinitesimally small distances with no genuine ultraviolet divergencies.

4. The paper

On October 1st 1954 the 32 years old Chen Ning Yang and somewhat younger Robert Mills published the paper [7] ***Conservation of Isotopic Spin and Isotopic Gauge Invariance*** in which they explored the possibility of requiring all interactions to be invariant under independent rotations of the isotopic spin at all space-time points. Taking the clue from QED they proposed that all physical processes (not involving electromagnetic field) be invariant under the space-time dependent isotopic gauge transformation of the form $\psi(x)' = S(x)\psi(x) = \exp(i\lambda(x))\psi(x)$, where λ is a traceless hermitian 2 matrix and ψ denotes the nucleon doublet. This requirement lead them directly to the Lagrangian that after quantization described a triplet (one neutral and two charged) of gauge bosons and their selfcouplings. The quanta of their so called b-field had spin unity and isospin unity. Note that in 1954 no vector meson was known with the first one to be discovered only in March 1961.

Yang and Mills represented graphically the elementary vertices for their b-field coupled to the nucleon field as well as the triple and quadrupole b-field selfcouplings and then turned attention to the important question of the mass of quanta of their b-field. Despite the fact that their Lagrangian contained no explicit mass term for the b-field, the authors were not sure whether this implies its masslessness:

"We next come to the question of the mass of the b-quantum, to which we do not have a satisfactory answer. One may argue that without a nucleon field the lagrangian would contain no quantity of the dimension of a mass and that therefore the mass of the b-quantum in such a case is zero. The argument is however subject to the criticism that, like all field theories, the b-field is beset with divergences and dimensional arguments are not satisfactory."

5. Under the spell of gauge principle

At the end of fifties Sakurai, Salam, Ward, Neeman, Glashow and others had explored the possibility that strong, weak as well as electromagnetic interactions can be generated by making local gauge transformations on the kinetic terms in the free Lagrangian for all particles.

For strong interactions the most straightforward extension of the idea of Yang and Mills was carried out by Salam and Ward [10] within the framework of the SU(3) version of the Sakata model. By gauging the fundamental baryonic triplet made of proton, neutron and Λ hyperon, Salam and Ward got the octet of selfinteracting gauge vector bosons and complete gauge invariant Lagrangian, to which they, however, added mass terms for baryons as well as gauge bosons.

Interestingly, exactly a month before Salam and Ward submitted their paper, Yuval Ne'eman, a young member of Salam's group at the Imperial College in London, submitted to Nuclear Physics his work ***Derivation of Strong Interactions from a Gauge Invariance*** [11] where the known baryons and pseudoscalar mesons are assigned to octets of SU(3) and the gauge invariance was imposed on their kinetic term. The resulting octets octet of SU(3) gauge bosons coincided with that of Salam and Ward. Ne'eman's paper is remarkable for clarity and straightforwardness with which the idea is put forward, taking into account that at beginning of 1961 still none of gauge bosons postulated by Ne'eman was known. Ne'eman's achievement is even more admirable if we realize that before 1958 he spent a decade serving in the Israeli Army.

6. To Eightfold way

On March 15, 1961, a month after Ne'eman submitted his paper to Nuclear Physics Gell-mann circulated an extensive preprint called ***The Eightfold Way: A Theory of Strong Interaction Symmetry*** [12], which contained basically the same idea of arranging known hadrons into octets of SU(3) flavor group as Ne'eman's paper. And similarly as the latter it also formulated their interactions explicitly in the framework of Yang-Mills theories. Their names and theory are mentioned on the very first page:

"The most attractive feature of the scheme is that it permits the description of eight vector mesons by a unified theory of the Yang-Mills type ...."

I cannot resist from citing several other statements, which illustrate the fact that Gell-mann's paper aimed at constructing complete dynamical theory based on the SU(3) flavor symmetry by imposing gauge invariance on octets of baryons and mesons:

"The vector mesons are introduced in a very natural way, by extension of the the gauge principle of Yang and Mills. Here we have a supermultiplet of eight mesons....

Now the vector mesons themselves carry F spin and therefore contribute to the current which is their source. The problem of constructing a nonlinear theory of this kind has been completely solved in the case of isotopic spin by Yang and Mills and by Shaw.

There are trilinear and quadrilinear interactions amongst the vector mesons, as usual ..."

The preprint also includes discussion of the properties of new vector mesons, like their decays, violation of SU(3) symmetry etc. In all, the paper is truly great but...

7. or not to Eightfold way?

Gell-mann never attempted to submit it for publication! As described in his biography [13], shortly after completing the preprint [12] he was beset by doubts about his proposal, some of these doubts stemming from (incorrect as it later turned out) data. What he sent for publication, and then twice revised before it finally appeared under the title ***Symmetries of Baryons and Mesons*** [14], was an extensive but very cautious discussion of the various approaches to symmetries and interactions of hadrons. The Eightfold way, the central notion of his preprint [12], appears first in Section VIII and only *"as an alternative to the symmetrical Sakata model.*

This caution may be understandable, but what is really striking is the total renouncement of the idea, central to the preprint [12], to describe the interactions between hadrons by means of gauge theory. The names of Yang and Mills are not even mentioned and their paper [7] is not quoted. Pseudoscalar as well as vector mesons (which, however, did not play role of gauge bosons) couple directly to baryons in a standard way and no attempt is made to compare such interaction term with that developed in [12] on the basis of gauge invariance. Interestingly, the second printing of this preprint was circulated after the appearance of the published paper [14], indicating that Gell-mann was hedging his position.

On the other hand we encounter in the publication [14] the phrase that has become standard part of most of Gell-mann's papers:

"Nowhere does our work conflict with the program of the Chew et al. of dynamical calculation of the S-matrix from strong interactions using dispersion relations"

supplemented by the statement that has set the general strategy of his thinking:

"If there are no fundamental fields and no CDD poles, all baryons and mesons being bound or resonant states of one another, models like Sakata will fail; the symmetry properties we have abstracted can still be correct, however."

8. Quarks with color: right degree of freedom to generate strong interactions

In early 1964 Gell-Mann and Zweig formulated the hypothesis that the Eightfold way reflected the existence, in whatever sense one might understand this word, of three fundamental building blocs of matter from which all hadrons were composed. Each of the three quarks, as Gell-mann called them, were soon endowed with another internal quantum, color. The historical and conceptual developments leading to quarks with flavour and colour have been reviewed in many articles, like, for instance, [15,16].

The crucial contribution of Yochiro Nambu was his idea that nature invented colour not so much to avoid the spin-statistics problem of the conventional quark model, but primarily to provide the appropriate degree of freedom to generate strong interactions between quarks (and gluons). In January 1965 he formulated a model [17] of colored quarks interacting via the exchange of an octet of colored vector bosons, which in nonrelativistic approximation reduced to the term analogous to spin-spin or isospin-isospin coupling in nuclear physics. This model, which had led to a gap between the lowest lying, color singlet, states corresponding to observed hadrons and color nonsinglet states higher (possibly infinitely) up, had all essential ingredients of QCD, but it was not a field theory.

9. But who really needs "real" quarks?

Although the idea of quarks as fundamental building blocs of hadrons was intuitively appealing, it took a decade for it to take firm roots and gain wide acceptance. In between it had to face serious objection due to the fact that despite serious efforts quarks had not been found in nature. In the early sixties, even before the formulation of the quark model, a new and truly radical doctrine was formulated by Geoffrey Chew. He rejected not only quantum field theory as the basic framework for description of strong interactions [18]:

"I believe the conventional association of fields with strong interacting particles to be empty. It seems to me that no aspect of strong interactions has been clarified by the field concept. Whatever success theory has achieved in this area is based on the unitarity of the analytically continued S-matrix plus symmetry principles...."

but also the very concept of elementary particles:

"The notion, inherent in conventional Lagrangian field theory, that certain particles are fundamental while others are complex, is becoming less and less palatable ..."

This radical doctrine, based more on faith than hard facts, has become an integral part of the approach adopted by Gell-mann and his collaborators towards the concept of quarks and the quark model. In the summer of 1967 Gell-mann expressed his view of the meaning of quark composition of hadrons in the following words [19]:

"The idea that mesons and baryons are made primarily of quarks is difficult to believe, since we know that, in the sense of dispersion theory, they are mostly, if not entirely, made up out of one another. The probability that a meson consists of a real quark pair rather than two mesons or a baryon and antibaryon must be quite small. Thus it seems to me that whether or not real quarks exist, the q *or* $\overline{q}$ *we have been talking about are mathematical entities ... If the mesons and baryons are made of mathematical quarks, then the quark model may perfectly well be compatible with bootstrap hypothesis, that hadrons are made up out of one another."*

To reconcile the doctrine of "nuclear democracy" with the fact that quarks were helpful in understanding the spectrum of observed hadrons, Gell-mann had to assume absolute confinement of quarks. In this approach, color confinement is an unavoidable consequence of the "nuclear democracy", whereas in QCD, as we understand it today, color confinement and the ensuing "nuclear democracy" are nontrivial consequences of the character of forces acting between colored quarks and gluons.

There was nothing wrong with this interpretation of the meaning of quarks and in 1967 this was a plausible approach. Nevertheless, further development, experimental as well as theoretical, vindicated Zweig's "naive" way of dealing with quarks, rather than the more abstract one adopted by Gell-mann.

10. Too much scaling may be misleading

The journey from rejection of quantum field theory for the description of strong interactions in the middle of fifties, to the discovery of asymptotic freedom and formulation of QCD in 1973, has been described by D. Gross in his excellent review [1] to which I have nothing to add. I merely wish to draw attention to one aspect of this story.

QCD has emerged to some extent as unwanted child of the attempts to explain the phenomenological success of the parton model, which was based upon the experimental observation of approximate scaling of nucleon structure functions. This experimental fact gave rise to two different schools of thought.

One, advocated by Gross, took the weak, but clearly observed scaling violations seriously and asked the question whether they could be accommodated in any quantum field theory. Realizing that only asymptotically free quantum field theories yield such behaviour led him and Wilczek to the fateful decision to try to close the last loophole - nonabelian gauge theories of Yang and Mills - in the then prevalent view that such quantum field theories do not exist. Their (and Politzer's) surprising discovery that under certain conditions Yang-Mills theories are asymptotically free changed the situation dramatically and led to the resurrection of quantum field theory and, in turn, this year Nobel Prize for physics.

Quite different conclusions were drawn from SLAC data by Bardeen, Fritzsch and Gell-mann. They did not regard quantum field theory as appropriate tool for the description of strong interactions, and interpreted SLAC data as argument against quantum field theory and in favour of exact scaling [20]:

"... one is considering the abstraction of results that are true only formally, with canonical manipulation of operators, and that fail, by powers of logarithmic factors, in each order of renormalized perturbation theory, in all barely renormalizable models. The reason for the recent trend is, of course, the tendency of the deep inelastic electron scattering experiments at SLAC to encourage belief in Bjorken scaling, which fails to every order of renormalized perturbation theory in barely renormalizable models."

Even five months after the discovery of asymptotic freedom the authors of [21] considered seriously the possibility that at very small distances the approximate Bjorken scaling will be replaced by exact (true) one:

"For us, the result that the color octet field theory model comes closer to asymptotic scaling than the color singlet model is interesting, but not necessarily conclusive, since we conjecture that there may be a modification at high frequencies that produces true asymptotic scaling."

This expectation proved wrong: parton model is not a short distance limit of QCD, but a good approximation to the latter at distances smaller that a fraction of fermi. The shorter the better, but at any finite distances the scaling is always only approximate.

11. Jets: crown witness of quarks and Yang-Mills dynamics

The concept that finally convinced most of the physicists about the physical reality of quarks and gluons are jets. These traces of quarks and gluons are now indispensable tool in almost all experimental investigations and their theoretical interpretations. By measuring jets and their properties we learn about the underlying dynamics of quarks and gluons. The fact that the latter are not observable as individual free objects complicates the analyses a bit, but not much as we now understand that the process of hadronization does not spoil the relation between quarks or gluons and jets. The close resemblance between jets on one side and leptons and gauge bosons on the other has been discussed by many authors, among them Frank Wilczek [23], whose picture is shown in Fig. 1b, flanked by a lego plot of a typical deep inelastic event as seen in H1 detector at HERA and a nice three jet event recorded by ALEPH detector at LEP. The way in which we now "see" quarks and gluons through their effects on our measuring instruments is not much different from the way we see electrons. I share the view of D. Gross [1]

"Nowadays, when you listen to experimentalists talk about their results they point to their lego plots and say, "Here we see a quark here a gluon". Believing is seeing, seeing is believing. We now believe in the physical reality of quarks and gluons ..." His words are

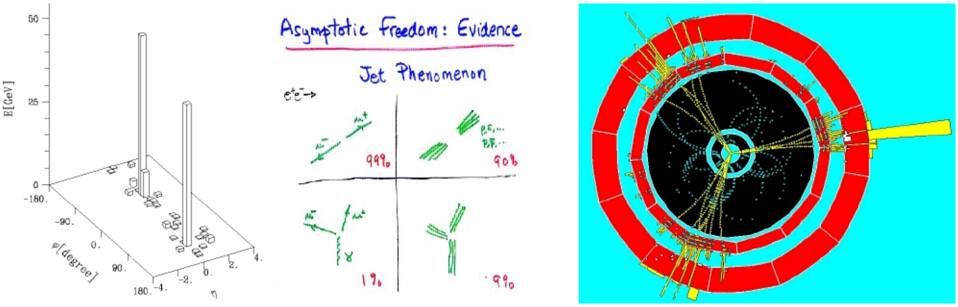

Figure 1. Two jet event as seen in H1 detector (left), the figure taken from [23] illustrating the relation between final state leptons and jets (middle) and three jet event in ALEPH.

nicely illustrated by the two events shown in Fig. 1a,c.

Detailed analysis of angular distribution of four jet events at LEP, like the one in Fig. 2, has been employed to test the basic feature of QCD as Yang-Mills theory, and namely the three gluon coupling. In Fig. 2 the QCD prediction for the distribution of the so called Bengtsson-Zerwas angle is compared to early L3 data as well as to the prediction of the theory with abelian gluon. The superiority of QCD description of data is clear.

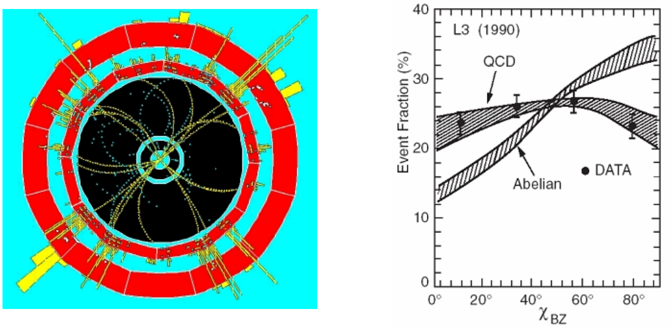

Figure 2. Four jet event recorded by ALEPH detector (left) and the distribution of Bengtson-Zerwas angle measured by the L3 collaboration at LEP .

Jet physics has also shown that the approach advocated by Fritzsch, Gell-Mann and other theorists and expressed most explicitly in [22]

"We assume here that quarks do not have real counterparts that are detectable in isolation in the laboratory they are supposed to be permanently bound inside mesons and baryons... It might be a convenience to abstract quark operators themselves, or other nonsinglets with respect to color, but it is not a necessity. It may not even be much of a convenience since

we would … be discussing a fictitious spectrum for each fictitious sector of Hilbert space, and we probably dont want to load ourselves with so much spurious information."

which led them to hope that

"We might eventually abstract from the quark vectorgluon field theory model enough algebraic information about the color singlet operators in the model to describe all the degrees of freedom that are present."

so that

"We would have a complete theory of the hadrons and their currents, and we need never mention any operators other than color singlets."

has not worked. It is now clear that if we want to understand the ever growing volume of data on hard processes, we must "load ourselves with spurious information" about colour nonsinglet quantities like quarks and gluons and, moreover, we have to take the subtle aspects of their dynamics, prescribe by QCD, seriously.

12. Pair of leaves: C.N. Yang and the role of mathematics in physics

The role of mathematics in the formulation of Yang-Mills theories has been widely discussed. The opinion of Yang on this point were expressed in an interview with D.Z. Zhang in [24]. Some excerpts (underlining by J. Ch.):

Q: How about ideas in mathematics becoming important for physics. We may recall Einstein was advised to pay attention to tensor analysis. Is that similar to your getting help from Simmons (an American mathematician who helped Yang with modern geometrical aspects of gauge theories)?

Yang: *As to the entry of mathematics into general theory of relativity and into gauge theory, the processes were quite different. In the former, Einstein could not formulate his ideas without Riemannian geometry, while in the latter, the equations were written down, but an intrinsic overall understanding of them was later supplied by mathematics.*

Q: Is it true what M.E. Mayer said in 1977: A reading of the Yang-Mills paper shows that the geometric meaning of the gauge potentials must have been clear to the authors since they use the gauge invariant derivative and the curvature for the connection ...

Yang: *Totally false. What Mills and I were doing in 1954 was generalizing Maxwells theory. We knew of no geometrical meaning of Maxwells theory and were not looking in this direction. Connection is a geometrical concept which I only learned around 1970.*

Q: An interesting question is whether you understood in 1954 the tremendous importance of your original paper ...

Yang: *No. In 1950 we felt our work was elegant. I realized its importance in the 1960s and its great importance to physics in the 1970s. Its relation to deep mathematics became clear to me only after 1974.*

Q: Is it important for a physicist to learn a lot of mathematics?

Yang: *No, if a physicist learns too much of mathematics, he or she is likely to be seduced by the value judgment of mathematics, and may loose his or her physical intuition. I have likened the relation between physics and mathematics to a pair of leaves. They share a small common part at the base, but mostly they are separate.*

Q: For a physicist, experimental results are more important to learn?

Yang: *This is right.*

13. Summary

The story of the emergence, acceptance and experimental confirmation of Yang-Mills theory of strong interactions shows that the fundamental building blocs of matter - quarks - must be taken seriously, not just as some auxiliary concept, and that there is no substitute for genuine dynamical laws governing the behaviour.

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