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Hermann Weyl and the Unity of Knowledge

John Archibald Wheeler

Hermann Weyl was -- is -- for many of us, and for me, a friend, a teacher, and a hero. A North German who became an enthusiastic American, he was a mathematical master figure to mathematicians, and to physicists a pioneer in quantum theory and relativity and discoverer of gauge theory. He lives for us today, and will live in time to come, in his great findings, his papers and books, and his human influence.

Among Weyl's papers is his Columbia University bicentennial lecture, "The Unity of Knowledge" (1). Unity? This world in which we live and have our being: what was it to Hermann Weyl? Not to him did the poet's words apply,

> Heaven wheels above you,
> Displaying to you her eternal glories,
> And still your eyes are on the ground.

The world to him was no dreary cavern; no, a miraculous panorama, exciting in him a passionate desire to capture its beauty, its order, and its unity. What can we learn from his search for unity?

Remarkable issues connected with the puzzle of existence confront us today in Hermann Weyl's domain of thought. Four among them I bring before you here as especially interesting: (1) What is the machinery of existence? (2) What is the deeper foundation of the quantum principle? (3) What is the proper position to take about the existence of the "continuum" of the natural numbers? And (4) what can we do to understand time as an entity, not precise and supplied free of charge from outside physics, but approximate and yet to be derived from within a new and deeper time-free physics? In brief, why time? What about the continuum? Why the quantum? What is existence?

Between four issues of such different flavor, can we hope at the end to see the glimmerings of a linkage? Let me anticipate. Let me propose that we shall find the unity-bringing theme in Weyl's own 1954 address on the unity of knowledge: "the realm of Being is not closed, but open."

Let us plot our course backward from our theme. The unity of knowledge? At the end. Just before the theme? Our consideration of those four great questions. Before the four questions? Some developments in knowledge relevant to those questions: contributions of Weyl; further findings since his day; and the driving force, straight out of the age of enlightenment, that impelled Weyl -- and thinkers before and since -- to apply reason to, and hope to make progress with, such great issues. And before this brief survey of some Weyl-related knowledge? A glimpse of Weyl himself.

The man

I last knew Weyl after I last knew him. Day after day in Zürich in late 1955 he had been answering letters of congratulation and good wishes received on his seventieth birthday, walking to the mailbox, posting them, and returning home. December eighth, thus making his way homeward, he collapsed on the sidewalk and, murmuring "Ellen," died. News of his unexpected death reached Princeton by the morning *New York Times.* Some days later our postman brought my wife and me Weyl's warm note of thanks. I like to think he sent it in that last mailing.

I first knew Weyl before I first knew him. Picture a youth of nineteen seated in a Vermont hillside pasture, at his family's summer place, with grazing cows around, studying Weyl's great book, *Theory of Groups and Quantum Mechanics* (2), sentence by sentence, in the original German edition, day after day, week after week. That was one student's introduction to quantum theory. And what an introduction it was! His style is that of a smiling figure on horseback, cutting a clean way through, on a beautiful path, with a swift bright sword.

Some years ago I was asked, like others, I am sure, to present to the Library of the American Philosophical Society the four books that had most influenced me. *Theory of Groups and Quantum Mechanics* was not last on my list. That book has, each time I read it, some great new message.

Erect, bright-eyed, smiling Hermann Weyl I first saw in the flesh when 1937 brought me to Princeton. There I attended his lectures on the Elie Cartan calculus of differential forms and their application to electromagnetism -- eloquent, simple, full of insights. Little did I dream that in thirty-five years I would be writing, in collaboration with Charles Misner and Kip Thorne, a book on gravitation, in which two chapters would be devoted to exactly that topic. At another time Weyl arranged to give a course at Princeton University on the history of mathematics. He explained to me one day that it was for him an absolute necessity to review, by lecturing, his subject of concern in all its length and breadth. Only so, he remarked, could he see the great lacunae, the places where deeper understanding is needed, where work should focus.

If I had to come up with a single word to characterize Hermann Weyl, the man, as I saw and knew him then and in the years to come, it would be that old fashioned word, so rarely heard in our day, "nobility." I use it here not only in the dictionary sense of "showing qualities of high moral character, as courage, generosity, honor," but also in the sense of showing exceptional vision. Weyl's eloquence in pointing out the peaks of the past in the world of learning and his aptitude in discerning new peaks in newly developing fields of thought surely were part and parcel of his lifelong passion for everything that is high in nature and man.

Weyl in the larger community of learning

Weyl belonged to the community of learning. He felt at home in the great wide world of thought. What that community is and what it means I first fully felt in Sunday morning walks through the woods around the Institute for Advanced Study with Hermann Weyl, Oswald Veblen, and two or three other colleagues from other disciplines. Topics ranged from art to the history of the Renaissance, from Helen Porter Lowe's current translation of Thomas Mann to Hella Weyl's putting Ortega y Gasset's Spanish into German and English, from mathematical logic to Europe's nightmare, from movements to men, and from ideologies to ideas. To pass the place where John von Neumann was developing his computer was to see Weyl's face light up with the delight of a small boy within touching distance of fireworks.

Weyl's new house was being built. Sometimes the walks took us there to inspect the progress and note the architecture. Outside, its shape and flavor were new to the Princeton community. Inside, when it was finished, order, calm, and beauty were the themes. From this place of happy reading, reflection, and writing Weyl's thought ranged out over Minerva's many meadows.

Not least for Hermann Weyl among the green pastures of the mind was art. His Vanuxem Lectures at Princeton University appear in that widely read and many times reprinted book, *Symmetry* (3). Nowhere more explicitly than here, in examples taken out of painting, symbolism, and the decorative arts, do we see the rich connections that his mind was forever making among art, history, science, and mathematics.

Art was for Weyl not only an activity practiced by others. Art characterized also his own way of thinking, writing, and speaking: art in the sense of bringing disparate themes into harmonious unity. The chapters in *Theory of Groups and Quantum Mechanics* masterfully alternate between group theory and quantum mechanics, between mathematical methods and their applications, between the ideas basic for groups and the concepts fundamental for physics. *Space Time Matter* (4) skillfully weaves together the deepest concepts of geometry with the physics of matter and motion. And did anyone ever bring together philosophy, mathematics, and physics into a happier unity than Weyl does in his famous book *Philosophy of Mathematics and Natural Science* (5)? The grace and sense of style characteristic of the whole book is epitomized in one of its best-known passages: "The objective world simply is, it does not *happen.* Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time."

If the painter-inventor moves things around on the screen of the mind to make a new and greater thing, if Henry Moore mentally abstracts a beautiful new form out of the whitened bones, ebony carvings, and rounded stones he chooses and lays before him each morning on his little worktable, so -- we can believe -- Hermann Weyl by like artistry knew how to select from his rich storehouse the ideas needed at the moment and push them about, this way and that, until they fell together in a new, greater, and more wonderful idea. Art for him was ever green.

Literature for him was another nourishing pasture of the mind, from Goethe and Gottfried Keller to Rilke and Mann and from Shakespeare and Coleridge to Ortega y Gasset and T. S. Eliot.

Through the green fields of history Weyl walked, too, with the greats, from Thucydides and Pliny to Vico and Ranke, Burckhardt, and Hesse. Surely it was from Weyl that I learnt that wonderful statement of Burckhardt, in *The Civilization of the Renaissance in* Italy, about the age-old human passion for superstition and astrology: “It is profoundly instructive to observe how powerless culture and enlightenment were against this delusion, since the latter had its support in the ardent imagination of the people, in the passionate wish to penetrate and determine the future.”

Weyl knew as well as anyone that we cannot know who we are and where we are going unless we know how we got here. “All history in the proper sense,” he reminded us, “is concerned with the development of one singular phenomenon: human civilization on earth” (1). What it takes to do history, he adds, is not science, not mathematics, not a measuring device, but interpretation. Interpretation “springs from the inner awareness and knowledge of myself. Therefore the work of a great historian depends on the richness and depth of his own inner experience.” How he would have applauded the motto of the distinguished historian Jack Hexter: “If you would read history, write history.” Participation is as necessary for understanding, Weyl recognized, as understanding is for participation.

In the field of economics it was enough, to satisfy Weyl's wish for understanding, to know Winfield Riefler and Oskar Morgenstern. The scene comes vividly to mind of a cocktail party where Weyl was a guest, and guests, too, were Morgenstern and von Neumann, working at that time on their great investigation, embodied in their book *Theory of Games and Economic Behavior.* Lively as was their participation in separate knots of guests, one or the other would sometimes break off, draw his fellow investigator aside, and report some new idea that had just bubbled up out of his subconscious. Then back to the party. Weyl himself, in such close touch with this newly developing realm of knowledge, found his own imagination caught up in it. He even published a paper on the subject, beginning with this sentence: “J. von Neumann's minimax problem in the theory of games belongs to the theory of linear inequalities and can be approached in the same elementary way in which I proved the fundamental facts about convex pyramids” (6).

Colleagues? Weyl knew that nobody can be anybody without somebodies around. Teatime was for him a central point of the day, an opportunity, as Oppenheimer later put it, to “explain to each other what we do not understand.” And cocktail parties? None do I remember as richer in interesting colleagues and lively conversation about fascinating subjects than those given by Hermann Weyl and his wife, Hella.

Weyl himself, so universally respected and beloved, at home in so many fields of thought, made one think of those words of Frederick II about Leibniz, in effect: “Founder of the Prussian Academy? He already was an academy!” Weyl was an academy.

For Weyl to update in 1947 his great 1927 survey of philosophy, mathematics, and natural science was an enormous undertaking, understandable only in the light of his passion for truth and unity. The marvelous discoveries in the thirty years since his death would surely have filled him with joy and reanimated his wish, harder now than ever for anyone to achieve, to capture for us all the larger unity of these findings.

Electricity and geometry before and after Weyl

What is the role of electricity in the geometry of spacetime? Weyl came back to this topic in paper after paper and book after book. The perspectives that he and his successors opened up are probably explored today, under the names of “gauge field theory” and “grand unified field theory,” by more investigators than there were in the entire physics community at the time of Weyl’s first paper in the field. Weyl invented the gauge concept in 1918. By 1928 he had reformulated and restated the idea in the way it is still understood today: “gauge invariance corresponds to the conservation of charge as a coordinate invariance corresponds to the conservation of energy-and-momentum” (7).

In 1950 Weyl referred to the 1921 idea of T. F. E. Kaluza that gauge invariance, might be connected with the possible presence in nature of a fifth dimension (8). He noted that O. Klein added to Kaluza's idea the further conception, arising out of quantum theory, that the fifth-dimensional part of the geometry is curled up into a very tight radius of curvature. This pregnant idea, students of modem particle theory know, has itself taken fifty years to bloom. Today the gauge field at each point in spacetime is envisaged as running, not around the one-dimensional rim of a tiny circle, but around an ultra-small cavity of much higher -- perhaps six units higher -- dimension. The variations of the field in the several directions describe not only the electromagnetic field but also the fields associated with neutrinos and all the rest of elementary particle physics. A particle mass itself corresponds in effect to an organ-pipe resonance of the geometry in this tiny world -- or, in mathematical terms, “fiber” -- attached to each point of spacetime. Weyl, were he still living, would rejoice in the rich modem development of elementary particle physics and be contributing himself, surely, to putting the theory of gauge fields and fiber bundles into a wider and deeper mathematical and physical framework.

Another insight Weyl gave us on the nature of electricity is topological in character and dates from 1924 (9). We still do not know how to assess it properly or how to fit it into the scheme of physics, although with each passing decade it receives more attention. The idea is simple. Wormholes thread through space as air channels through Swiss cheese. Electricity is not electricity. Electricity is electric lines of force trapped in the topology of space.

Niels Bohr, presented at one point with this thought, immediately asked, “but will not any wormhole pinch itself off?” Subsequent calculations showed that, in the context of classical theory, indeed it will (10). Further consideration, however, has made it clear that one should deal not with classical wormholes but with quantum fluctuations in the geometry of space, wormholes at the unbelievably small Planck scale of distances (on the order of 10^{-33} cm), giving a “foamlike” structure to space with these microscopic wormholes all the time and everywhere being created and annihilated, annihilated and created (11, 12, 13, 14).

How are we to assess Weyl’s proposal, thus updated? That is a question not for today but for tomorrow. To deal with it is to work at the frontier between quantum theory and general relativity, in that realm often called “quantum geometrodynamics,” one of the most challenging fields of research of our day.

General relativity and gravitation

It is sometimes said today that no progress has been made in uniting general relativity and quantum theory. No statement could be further from the truth. We have possessed for years the appropriate wave equation. We have also known, through the work of Uhich Gerlach (15), that the predictions of this wave equation go over, in the so-called correspondence-principle limit, into those of classical geometrodynarnics. In the past year we have seen applications; of this wave equation, by James Hartle and Stephen Hawking (16), to important issues of cosmology. But this whole field of investigation is so new and strange that we have still some way to go before we have a proper notion what to make of Weyl’s idea that “electricity is field trapped by topology.” But disregard it? Only at our peril.

In astrophysics generally and cosmology in particular a fantastic growth has taken place in the years since Weyl’s death, thanks to the existence of redshifts in stellar spectra (which show -- he was the first truly to explain -- that the universe is expanding) and a marvelous armament of telescopes, of not one kind alone, but four: x-ray, optical, infrared, and radio. Among all developments in astrophysics, surely none would have gripped his imagination more than the black hole, a star that has undergone complete gravitational collapse.

Already in the earliest days of “gravitation-as-geometry” Weyl taught us a useful new view of a “mass.” (4) Draw a sphere around it. Stretch this sphere out in time into a fattened world line, a world tube. What happens to a star duster, a whole collection of masses interacting gravitationally with one another? Viewed in spacetime in Weyl’s way, it becomes a twisting, writhing pattern of world tubes. To predict that pattern, he explained, we don't have to look inside the tubes.

Weyl’s way of looking at mass has a special relevance for a black hole -- itself invisible -- that happens to be paired with a visible companion. Weyl would be delighted today to find that the black hole is not simply an object on pencil and paper, that it really exists. We already have striking evidence for two black holes in the range of 10 to 20 times the mass of the sun (17, 18). Each is invisible itself. Each by its powerful pull swings about it in a tight, swift orbit a normal star which we can and do see. We see also x-rays. They do not come from the black hole itself. They come from gas spewed out onto the black hole from its normal neighbor. The inward-streaming gas is crunched up to high density, and therefore enormous temperature, by the powerful pull of the completely collapsed object. Hence the x-rays. They fluctuate in intensity from millisecond to millisecond because of random variations in the density of the gas falling in, as the smoke coming out of a factory chimney fluctuates in its darkness from second to second. In addition to those two black holes of stellar mass there also exists at the center of the Milky Way, according to ever stronger evidence (19), a black hole with mass about three and a half million times the mass of the sun.

Never has curved, empty space come more spectacularly to man’s attention than it does in black hole physics. Never has any branch of physics been developed more fully and more richly on the basis of purely geometric reasoning. And never more insistently than in black hole physics have we been driven to the very frontiers between gravitation theory and quantum theory.

From black hole and quantum to “information has mass”

Jacob Bekenstein found himself forced in 1973 to conclude that the surface area of the so-called horizon of a black hole not only is analogous to entropy, it is entropy; the surface gravity not only is analogous to temperature, it is temperature (20). This conclusion seemed so preposterous to Brandon Carter and Stephen Hawking that they set out to disprove it. Along the way Hawking discovered that marvelous process (21), now known by his name, through which a black hole -- indeed endowed after all with a Bekenstein temperature -- can evaporate particles and radiation (less than one watt, however, from a black hole with the mass of the sun, and less in proportion as the mass is greater).

Black hole physics has led to one great discovery that has in it not one word of black hole physics. I refer to the by now famous formula of William Unruh (22). It tells us that an accelerated detector, located in cold empty space, will nevertheless, by reason of that very acceleration, experience and register a temperature, a temperature proportional to the product of Planck's quantum constant and that acceleration. This result generalizes Bekenstein's conclusion about the temperature at the "surface" of a black hole.

Unruh's formula ties together three great domains of physics. One is relativity. The second is quantum theory. The third, heat physics or thermodynamics or statistical mechanics, is also at bottom a part of information theory. I know no more beautiful discovery in recent years, nor any that connects more instructively three fields of endeavor dear to Weyl's thinking and writing.

A brave new proposal of Bekenstein is now the subject of exploration by more and more investigators (23): that there is no device whatsoever that will store a given number of bits of information which does not have a product of mass and linear dimensions expressed in appropriate units-which is at least as great as that number of bits. Information is not dreamlike nothingness. What an incentive to put "information" into the center of our thinking about physics! And to ask, is information theory the foundation for all we see and know?

From genes to DNA and from evolution to teleology

Information theory, Weyl knew, is central to the gene, the machinery of life, and evolution. "The mighty drama of organic evolution," as Weyl called it, was for him no domain of thought to be left to a few specialists, but a vital topic of concern to every thinking person. He gave it an important place in 1947 when he updated his *Philosophy of Mathematics and Natural Science.* "The decisive point," he suggests, "is perhaps this: when one deals with complex molecules consisting of something like a million atoms, the manifold of possible atomic combinations is immensely larger than those actually occurring in nature." Combinations capable of functioning as genes are extremely rare and can be "found" only by a selective process that probes many possibilities and uses previously conquered positions as bases for further advance, slowly groping its way from simple to more complicated structures. "But this formulation of the problem," he adds, "does not give more than the vaguest hint for its solution" (24).

How Weyl would have delighted in the DNA Of Francis Crick and James Watson. How fascinated he would have been by the "life machine" of Manfred Eigen and his Göttingen colleagues, showing as it does the stupendous variety of life forms that can develop, and emphasizing the role of what we can only call "accident" in deciding which shall come into being. How eagerly he would have studied those twin discoveries of John J. Hopfield: nature's method of "proofreading" molecules and DNA, without which life would quickly end in disaster; and insights into how the coupling of neuron to neuron powers memory and memory search (25, 26).

In what way did life begin? Weyl describes the idea of A. I. Oparin that organic molecules, formed by chance processes and accumulated in favorable spots, in time built up -- in the absence of enzymatic breakdown -- to the concentration at which self-duplicating molecules could form (27). He would have been interested in the proposal that droplets were the centers of accumulation and the place of origin of life (28), and in the theory, more widely studied today, that it was day which served as the organizing material (29).

The concept that life fills every ecological niche was not new to Weyl; but how interested he would have been in the 1978 finding of those students of evolutionary history, V. Salvini-Plawen and Ernst Mayr, that the eye -- the "window of the mind" -- originated over and over again, independently, at least forty times (30). What would he have said to the thesis of Homer Smith, so vividly expressed in his Pulitzer Prize-winning book, Kamongo, that man himself may, from the standpoint of the future development of the evolutionary tree, mark a blind alley of life? In contrast, Weyl himself, discussing teleology, remarks that "the temptation of an interpretation in terms of an overall plan of evolution is almost irresistible" (31). Could he have known that the distinguished physiologist Christian Bohr, father of Niels Bohr, while wholeheartedly accepting and supporting the Darwinian theory of evolution at a time when that position was not popular, nevertheless also felt that evolution, understood deeply, would prove to be compatible with ultimate purpose?

Teleology, come into physics? How? “Is it conceivable,” Weyl asks, “that immaterial factors having the nature of images, ideas, ‘building plans,’ also intervene in the evolution of the living world as a whole?”

Bohr's great smoky dragon

The question about teleology that Weyl put to himself we have more temptation than ever to put to ourselves in this day, when the deeper lessons of quantum theory are so widely perceived.

The idea is old that the past has no existence except in the records of today. In our time this thought takes new poignancy in the concept of Bohr’s elementary quantum phenomenon and the so-called delayed-choice experiment (32). Ascribe a polarization, a direction of vibration, to the photon that began its journey six billion years ago, before there was any Earth, still less any life? Meaningless! Not until the analyzer has been set to this, that, or the other specific chosen orientation; not until the elementary quantum phenomenon that began so long ago -- and stretches out, unknown and unknowable, like a great smoky dragon through the vast intervening reach of space and time -- has been brought to a close by an irreversible act of amplification; not until a record has been produced of either “yes, this direction of polarization” or “no, the contrary direction of polarization”; not until then do we have the right to attribute any polarization to the photon that began its course so long ago. There is an inescapable sense in which we, in the here and now, by a delayed setting of our analyzer of polarization to one or another angle, have an inescapable, an irretrievable, an unavoidable influence on what we have the right to say about what we call the past.

This circumstance is one of the more striking information-theoretic aspects of what we call existence. Moreover, the delayed-choice elementary quantum phenomenon is well established by theory and, within the last few years, by experiment (33, 34, 35). Any view of existence which does not reckon with quantum theory and the elementary quantum phenomenon is medieval. Quantum theory marks the summit of the exact natural science of our day -- that “exact natural science” which, Weyl reminds us, “is the most distinctive feature of our culture” (36).

Intellectual antecedents

The man who ranged so far in his thought had mathematics as the firm backbone of his intellectual life. Distinguished as a physicist, as a philosopher, as a thinker, he was above all a great mathematician, serving as professor of mathematics from 1913 to 1930 at Zurich, from 1930 to 1933 at Göttingen, and at the Princeton Institute for Advanced Study from October 1933 to his retirement. What thinkers and currents of thought guided Weyl into his lifework: mathematics, philosophy, physics?

“As a schoolboy,” he recalls, “I came to know Kant’s doctrine of the ideal character of space and time, which at once moved me powerfully” (37). He was still torturing himself, he tells us, with Kant’s *Schematismus der reinen Verstandesbegriffe* when he arrived as a university student at Göttingen. That was one year before special relativity burst on the world. What a time to arrive, just after David Hilbert, world leader of mathematics, had published his *Grundlagen der Geometrie,* breaking with Kant’s predisposition for Euclidean geometry and taking up, in the great tradition of Karl Friedrich Gauss and Bernhard Riemann, the construction and properties of non-Euclidean geometries, and-having just published an important book on number theory, *Zahlbericht* -- was giving absorbing lectures on that field of research. Philosophy! Mathematics! Physics! Each was sounding its stirring trumpet blast to an impressionable young man. Mathematics, being represented in Göttingen by its number one man, won the number one place in Weyl’s heart.

Allow here a pause for a brief song of thanksgiving. Shall we dedicate it to that Prince of Hanover whom the English-speaking world knows as George II? Or shall we praise instead the advisers at whose instance that ruler initiated, within a few years of each other, two now famous communities of learning? Those centers, both precious to Hermann Weyl, are known today as Göttingen (1734) and Princeton (1746). The advisers, blessed be their names-Christian Wolff, the disciple of Leibniz in the German-speaking world, and a group of concerned men of learning in the colony of New Jersey had for the two schools a common purpose: to testify to the glories of creation by looking at them, investigating them, and teaching about them.

That goal transmutes itself into ever new language with the arrival of ever new generations, In our century this powerful Göttingen-Princeton tradition, springing straight out of the age of enlightenment, has nourished deep learning -- a happy, intense, livelong-day search by great thinkers for beauty, order, and understanding.

What way of work did Weyl adopt? Three ways we know to make advances: the ways of the mole, the mutt, and the map. The mole starts at one point in the ground and systematically goes forward. Great science has been done by people so guided. The mutt sniffs around and is led on from one due to another. Great physics is done that way. But the third method of advance is marked by the mapmaker, the philosopher who conceives the overall picture, has a feeling for how things fit together, and finds his way, by that sense of fitness, to where new truth lies. That was Weyl.

To Kant, to the Göttingen of Gauss and Riemann, and to Weyl the number one example of mapmaker, of philosopher, of guide in the enterprise of discovery was Leibniz. "Among the heroes of philosophy," Weyl tells us, "it was Leibniz above all who possessed a keen eye for the essential" (38). He goes on to remind us that "Leibniz planned to span Europe by a network of academies, centers of research which he expected to become a strong combine for the promotion of enlightenment" (39). Hermann Weyl and his longtime Princeton colleague, Kurt Gödel, shared an interest in Leibniz' notes on his project of a *characteristica universalis.* Leibniz hoped for a system for discovering truth in all the great wide world of thought comparable in its power to the method that Newton had brought to physics. Leibniz argued that such a tool of thought, such a method, such a philosopher's stone, once discovered, would be so powerful that it could be entrusted only to young people of the highest moral character.

If philosophy, the map, displayed the goals, mathematics -- in the shape of Hilbert -- showed the arriving Göttingen student the way. Weyl tells us the impression made upon him by Hilbert's irresistible optimism, "his spiritual passion, his unshakable faith in the supreme value of science, and his firm confidence in the power of reason to find simple and clear answers to simple and clear questions." No one who in his twenties had the privilege to listen to Weyl's lectures can fail to turn around and apply to Weyl himself those very words. Neither can anyone who reads Weyl, and admires his style, fail to be reminded of Weyl's own writing by what he says of the lucidity of Hilbert: "It is as if you are on a swift walk through a sunny open landscape; you look freely around, demarcation lines and connecting roads are pointed out to you before you must brace yourself to climb the hill; then the path goes straight up, no ambling around, no detours" (40).

Electrified by Leibniz and Kant, and under the magnetic influence of Hilbert, Weyl leaped wholeheartedly, as he later put it, into "the deep river of mathematics." That leap marked the starting point of his lifelong contributions to ever widening spheres of thought.

Out of Weyl's thinking, out of his speaking, out of his writing -- and out of work since his day -- what guidance can we now discover on our quartet of questions: the mechanism of existence, the origin of the quantum, the problem of the continuum, and the deeper foundations of the idea of time?

The four mysteries

Existence, the preposterous miracle of existence! To whom has the world of opening day never come as an unbelievable sight? And to whom have the stars overhead and the hand and voice nearby never appeared as unutterably wonderful, totally beyond understanding? I know no great thinker of any land or era who does not regard existence as the mystery of all mysteries.

But is the quantum a mystery, too? We know that the way the quantum theory works is no mystery. It is expounded in a hundred texts. But from what deeper principle does its authority and its way of action derive? What central concept undergirds it all? Surely the magic central idea is so compelling that when we see it, we will all say to each other, "Oh how simple, how beautiful! How could it have been otherwise? How could we have been so stupid so long?" But what is the decisive clue to it all that we of today are missing?

The continuum of natural numbers: who could dispense with them who works with matter and motion, particles and fields, space and time? Indeed, as Weyl reminds us, "Classical analysis, the mathematics of real variables as we know it and as it is applied in mathematics and physics, has simply no use for a continuum of numbers of different levels [integers, rational fractions algebraic numbers, etc.]." Yet, he goes on to say, "[L. E. J.] Brouwer made it clear, as I think beyond any doubt, that there is no evidence supporting the belief in the existential characters of the totality of all natural numbers." More generally, he adds, "belief in this transcendental world [of mathematical ideals, of propositions of infinite length, and of a continuum of natural numbers] taxes the strength of our faith hardly less than the doctrines of the early Fathers of the Church or of the scholastic philosophers of the Middle Ages" (41). Then how can physics in good conscience go on using in its description of existence a number system that does not even exist?

Time? The concept did not descend from heaven, but from the mouth of man, an early thinker, his name long lost. Time today is in trouble. Time ends in a big bang and gravitational collapse. Quantum theory denies all meaning to the concepts of before and after in the world of the very small. Most of all, time has not yet been brought into submission to the rule of physics. It is fed in from outside physics. It has someday to be derived from inside physics -- when physics is deep enough to measure up to the task.

Four puzzles? Four dues. Shall we look at them one by one?

Existence: The anthropic principle

Let us begin on puzzle number one, existence, with what sounds at first hearing as a far-away note, a solitary and pregnant passage from a 1919 paper of Hermann Weyl on general relativity. There he points out the first of the by now famous large-number coincidences of physics in a single sentence, which we can be forgiven for taking apart into three: "It is a fact that pure numbers appear with the electron, the magnitude of which is totally different from one. For example, the ratio of the electron radius to the electron's gravitational radius is of the order of 10^{40}. The ratio of the electron radius to the world radius may be of similar proportions" (42).

This coincidence between two enormous numbers of very different origin was called in a paper twelve years later by Fritz Zwicky "Weyl's hypothesis" (43). But not everybody reads the literature. Later it and further such relations were called "Eddington's large number coincidences" (44), then, later, "Dirac's large numbers" (45), and so they are widely known today. But it all began with Weyl.

What feature of nature lies behind these large-number coincidences? What fixes these and other dimensionless constants of physics? Advocates of anthropic principle, nowadays investigated by more and more physicists and astrophysicists, propose a perspective-shattering answer: not only is man adapted to the universe, the universe is adapted to man. Imagine a universe in which one or another of the fundamental dimensionless constants of physics differs from this world's value by a few percent one way or the other. The consequences for the physics of stars so multiply themselves up -- according to the analyses of numerous investigators (46) -- that man could never have come into being in such a universe.

The anthropic principle superficially looks like a tautology: we're adapted to the universe because we're adapted to the universe. However, closer study by Brandon Carter (47) shows that the idea leads to an amazing, concrete, and someday testable prediction: it sets a limit of a few hundred million years, at most, on the time that the earth will continue to be an inhabitable planet. This prediction is derived from our knowledge of evolutionary biology and from modem statistical analysis. A simple mathematical expression, called Carter's inequality, relates the likely duration of life on Earth in the future to the number of improbable evolutionary steps required in the past for the emergence of intelligent life.

Is the machinery of the universe so set up, and from the very beginning, that it is guaranteed to produce intelligent life at some long-distant point in its history-to-be? And is this proposition testable by the Carter prediction? Perhaps. But how should such a fantastic correlation come about between big and small, between machinery and life, between future and past?

Some who investigate the anthropic principle put forward the notion of an ensemble of universes, distinguished one from another by different values of the dimensionality and the dimensionless constants of physics. In the overwhelming majority of cases, they argue, intelligent life is and always will be impossible. We belong, on this view, to one of the rare exceptions, a universe where awareness can and does develop.

We can reject some of these ideas without rejecting everything. We can forgo the notion of an ensemble of universes as outside the legitimate bounds of logical discourse. We can nevertheless examine the anthropic principle itself as an attractive working hypothesis attractive because it exposes itself, by its predictive power, to destruction in the sense of Karl Popper (48) and because it makes sense out of numbers that would otherwise have no rationale. But without multitudes of universes to experiment on, to bungle and to ruin à la David Hume, with solely this one and only universe to work with, how can history ever have made things come out right, ever given a world of life, ever thrown up a communicating community of the kind required for the establishment of meaning? In brief, how can the machinery of the universe ever be imagined to get set up at the very beginning so as to produce man now? Impossible! Or impossible unless somehow -- preposterous idea -- meaning itself powers creation. But how? Is that what the quantum is all about?

To ask this question is to look at the puzzle of existence from a new perspective; to see a thread of connection with puzzle number two, the how-come of the quantum.

Machinery, law, quantum

Machinery of existence for us means laws of physics under the overarching governance of the quantum principle; in brief, laws and the quantum. How can the quantum ever be understood as powered by meaning? Or laws of physics, by meaning?

Weyl reminds us that "postulation of the external world does not guarantee that such a world will arise ... from the phenomena ... through the cognitive work of reason ... which attempts to create concordance. For this to take place," Weyl emphasizes -- updating Kant's concept of reality as "that which is connected with perception according to laws" -- "it is necessary that the world be governed throughout by simple elementary laws" (49).

Those laws, so beautiful, so necessary for an understandable, meaningful world, and on first inspection so full of structure, turn out every one of them on closer look to be built in large measure on tautology, mathematical identity, the most elementary statement of algebraic geometry: the principle that the boundary of a boundary is zero. Electromagnetism, in the shape of Maxwell's equations, seen in four-dimensional perspective, falls apart into two divisions, one equivalent to the statement that the one-dimensional boundary of the two-dimensional boundary of a three-dimensional region is identically zero; the other, that the two-dimensional boundary of the three-dimensional boundary of a four-dimensional region likewise identically vanishes. The concept of the vanishing of the boundary of a boundary both in its 1-2-3 and in its 2-3-4 forms is used again in gravitation physics and yet again in the Yang-Mills or chromodynamic or string theory -- of one or another degree of sophistication -- of elementary particle physics. How strange, we say at first. And then we ask ourselves, how could it have been otherwise?

Surely -- big bang and gravitational collapse advise us -- the laws of physics cannot have existed from everlasting to everlasting. They must have come into being at the one gate of time, must fade away at the other. But at the beginning there were no gears and pinions, no corps of Swiss watchmakers to put things together, not even a pre-existing plan. If this assessment is correct, every law of physics must be at bottom like the second law of thermodynamics, higgledy-piggledy in character, based on blind chance. Physics must be in the end law without law. Its undergirding must be a principle of organization which is no organization at all. In all of mathematics, nothing of this kind more obviously offers itself than the principle that the boundary of a boundary is zero. That this principle should pervade physics, as it does -- is that the only way that nature has to signal to us a construction without a plan, a blueprint for physics that is the very epitome of austerity?

No. A second sign directs the seeker for the plan of existence still more clearly to austerity: the quantum.

What is the thread that connects mystery number two, the quantum, with puzzle number one, the machinery of existence?

Does the very concept of existence imply that there must be a world sitting "out there"? That was the view of many a great thinker before the advent of quantum theory -- and of Einstein himself to the end of his days. Nothing made him more unhappy than the thought that the observer-participator has anything to do with the establishment of what one is accustomed to call reality. In the last talk he ever gave, some months before his death, to my seminar on relativity, he explained how he had come to relativity and what relativity meant to him, but went on to express his discomfort with quantum theory: "If a person, such as a mouse, looks at the universe, does that change the state of the universe?" And to the visitor defending quantum theory to him in his study he objected against its probability features in the words, "God does not play dice" (50).

Weyl, in contrast, spoke up for the physics community when he stated, "Quantum theory is incompatible with the idea that a strictly causal theory of unknown content stands behind it.... The reasons for the passage from classical to quantum physics are no less compelling than those for the relinquishment of absolute space and time by relativity theory; the success, if measured by the empirical facts made intelligible, is incomparably greater" (51).

No one saw deeper into the central point of quantum theory than Niels Bohr, lifelong friend of Hermann Weyl; and no one stated its importance more strongly than he in his last interview, a few hours before his unexpected death: "They [certain philosophers] have not that instinct that it is important to learn something and that we must be prepared to learn something of very great importance ... They did not see that it [the complementary description of nature as it is seen in quantum theory] was an objective description-and that it was the only possible objective description" (52).

Only possible? There is not a single sight, not a single sound, not a single sense impression which does not derive in the last analysis from one or more elementary quantum phenomena.

Objective? Not until the observing sense, or observing device -- by its geometry, its layout, and its adjustment -- has chosen the question to be asked, and by its registration has made a record long enough lived to produce internal or external action, has an elementary quantum phenomenon taken place that contributes to the formation of what we call reality. No other way do we know to build this reality. Existence? How else is it brought into being except through elementary quantum phenomena?

Unbelievable! The number of bits of information that anyone can accumulate in a lifetime is incredibly small compared to the richness we know to be there in the great wide world. Even if we include more than a single observer-participator and count up the contribution of every member of the meaning-establishing community, of every observer-participator past, present, and future, what hope is there for deriving existence out of quantum acts? Of them there is at most a countable infinity. In contrast, existence seems to present us everywhere with continuous infinities: a continuous infinity of locations for particles, a continuous infinity of field strengths, a continuous infinity of degrees of freedom of dynamic space geometry.

Issue number two, the how-come of the quantum, here shows some thread of connection with issue number three, the continuum. How close is that connection?

The continuum

The continuum of natural numbers, Weyl taught us, is an illusion. It is an idealization. It is a dream. With numbers of ever increasing mathematical sophistication we can approach that infinity ever more closely; but we commit a folly if we think we can ever get there. That, in poor man's language, is the inescapable lesson of Gödel's theorem and modem mathematical logic.

Do we not have to say that the notion of a physical world with a continuous infinity of degrees of freedom is an equal idealization, an equal folly, an equal trespass beyond strict logic?

Do we not do better to recognize that what we call existence consists of countably many iron posts of observation between which we fill in by an elaborate papier-mâché construction of imagination and theory? The artist paints in the faces of five angels, with diminishing size, followed by a row of dots of still further decreasing size, stretching out into a line that runs to the horizon; but the beholder believes himself to see an infinitude of angels. When Bohr tells us that quantum theory gives us the only objective description of nature of which one can possibly conceive, is he not also telling us that no description can make sense which is not founded upon the finite?

For the advancing army of physics, battling for many a decade with heat and sound, fields and particles, gravitation and spacetime geometry, the cavalry of mathematics, galloping out ahead, provided what it thought to be the rationale for the natural number system. Encounter with the quantum has taught us, however, that we acquire our knowledge in bits; that the continuum is forever beyond our reach. Yet for daily work the concept of the continuum has been and will continue to be as indispensable for physics as it is for mathematics. In either field of endeavor, in any given enterprise, we can adopt the continuum and give up absolute logical rigor, or adopt rigor and give up the continuum, but we can't pursue both approaches at the same time in the same application.

Adopt rigor or adopt the continuum? These ways of speaking should not be counted as contradictory, but as complementary. This complementarity between the continuum and logical rigor we accept and operate with today in the realm of mathematics. The hard-won power thus to assess correctly the continuum of the natural numbers grew out of titanic struggles in the realm of mathematical logic in which Hermann Weyl took a leading part (53). The level of synthesis achieved by now in mathematics is still far beyond our reach today in physics. Happily the courageous outpost-cavalry of mathematical logic prepares the way, not only for the main cavalry that is mathematics, but also for the army that is physics, and nowhere more critically so than in its assault on the problem of existence.

Time

Time, among all concepts in the world of physics, puts up the greatest resistance to being dethroned from ideal continuum to the world of the discrete, of information, of bits. Of physics the heart is dynamics, and of dynamics the heart is time. That time parameter we treat today, however, as provided for us free of charge from outside, as our forebears regarded elasticity a hundred years ago. In our day we have learned that there is no such thing as elasticity in the space between the electron and the nucleus. Elasticity, thanks to solid state physics, has been reduced from primordial and precise to secondary, approximate, and derived. Time today requires a like reduction.

Reduce time? The idea of reduction is old. Weyl reminds us that "the doctrine of the subjectivity of sense qualities has been intimately connected with the progress of science ever since Democritus laid down the principle, 'Sweet and bitter, cold and warm, as well as the colors, all these things exist but in opinion and not in reality; what really exist are unchangeable particles, atoms, which move in empty space' " (54). In accordance with this view of Democritus, we understand green today as a characteristic frequency of 5.7×10^{14} vibrations per second; freezing, a characteristic energy of 3.7×10^{-14} g cm^2/sec^2; and a detectable sound, an air pressure amplitude of 10^{-3} g/cm sec^2 -- motions indeed in empty space.

But time: how is time to be reduced to more primitive concepts? Reduced from the continuum to something built on bits? And along with the reduction of time how are we to understand that puzzling conservation of "I" from decade to decade, so vividly expressed by Weyl in his quotation from *Der Rosenkavalier,* where the marshal's wife looks into her mirror and asks, "How can it really be that I was little Resi -- and that I also one day will be old woman?" (55).

Of all obstacles to a thoroughly penetrating account of existence, none looms up more dismayingly than "time." Explain time? Not without explaining existence. Explain existence? Not without explaining time. To uncover the deep and hidden connection between time and existence, to close on itself our quartet of questions, is a task for the future.

More problems? More clues, more hope

A great problem, we know, means great hope, according to that time-honored doctrine of "no discovery without a paradox." A still better formula has emerged out of the science of this century: no great advance without a double mystery, a double paradox, a double problem, two dues that can be played off against each other to yield the answer. Fortunate are we to have before us two such mighty mysteries as time and existence -- each linked with two other great questions, the quantum and the continuum.

Last year was the hundredth birthday not only of Hermann Weyl but also of Niels Bohr. Their double drumbeat summons us to a great undertaking, tells us that we can and must achieve four victories:

Understand the quantum as based on an utterly simple and -- when we see it -- completely obvious idea.

Explain existence by the same idea that explains the quantum.

Through this larger vision of existence and the quantum, recognize that the continuum of that physical world out there and the bit-by-bit means by which alone we can define that world are not contradictory, but complementary.

Reduce time into subjugation to physics.

As we face these stirring challenges, a warm memory gives us courage. Hermann Weyl has not died. His great works speak prophesy to us in this century and will continue to speak wisdom in the coming century. If we seek a single word to stand for the life and work of Hermann Weyl, what better word can we find than passion? Passion to understand the secret of existence was his, passion for that dear, luminous beauty of conception which we associate with the Greeks, passionate attachment to the community of learning, and passionate belief in the unity of knowledge.

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